PERIÓDICO TCHÊ QUÍMICA
ARTIGO ORIGINAL

IDENTIFICAÇÃO DO ALVO DE DISPERSÃO COM BASE NA FUNÇÃO RADIAL DE REDES NEURAIS ARTIFICIAIS NA PRESENÇA DE RUÍDO NÃO ESTACIONÁRIO

SCATTERING TARGET IDENTIFICATION BASED ON RADIAL BASIS FUNCTION ARTIFICIAL NEURAL NETWORKS IN THE PRESENCE OF NON-STATIONARY NOISE

РАСПРЕДЕЛЕНИЕ ЦЕЛЕВОЙ ИДЕНТИФИКАЦИИ НА ОСНОВЕ РАДИАЛЬНОЙ ФУНКЦИИ ИСКУССТВЕННЫХ НЕЙРОННЫХ СЕТЕЙ В ПРИСУТСТВИИ НЕСТАЦИОНАРНОГО ШУМА

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RESUMO

O artigo discute o problema do reconhecimento de alvos de radar realizado em imagens de radar complexas. Uma abordagem de rede neural artificial (ANN) com uma função de base radial (RBF) é proposta para identificar dispersadores de pontos localizados em uma imagem de radar. O conceito atualizado de blocos adaptativos simples como base para a montagem da rede permite o desenvolvimento de um esquema de extração de recursos baseado em ANN para processamento bidimensional de sinais. Foi demonstrado que ANN que implementa unidades de processamento neural de RBF pode ser usada para identificar alvos de radar descritos por um conjunto de dispersores individuais mesmo quando a distância relativa entre os dispersores é comparável ou menor que a largura efetiva de cada dispersor. A abordagem apresentada neste artigo foi a utilização da rede neural com a função de base radial especialmente sintetizada (RBF), usada para aproximar imagens de radar selecionadas transmitidas à sua entrada. Os resultados obtidos indicaram uma alta precisão na estimativa de centros individuais de dispersores na presença de ruído, o que não se limita ao caso estacionário, mas é considerado ciclostacionário. Também foi mostrado que os parâmetros que descrevem as coordenadas dos centros de dispersão podem ser extraídos com sucesso da ANN treinada após cerca de cem épocas passadas no processo de treinamento da ANN, que é realizado usando o método de descida com gradiente modificado. O principal resultado foi uma demonstração da possibilidade de usar redes neurais para análise automática de imagens de radar, que é parte integrante do conjunto de tarefas que formam o problema do reconhecimento de alvos. O algoritmo proposto implementa uma abordagem para identificar sistemas feitos usando procedimentos de treinamento em redes neurais.

Palavras-chave: *classificação do difusor, não estacionário, ciclo-estacionário, imagem de radar integrada, alvos de radar.*

ABSTRACT

The paper deals with the radar target discrimination problem performed on complex radar images. The approach based on radial basis function (RBF) artificial neural network (ANN) is proposed for the identification of point scatterers placed within a radar image. The renewed concept of simple adaptive units as the foundation for network assembling allows one to design an ANN-based feature extraction scheme for the two-dimensional signal processing. It was shown that ANN implementing RBF neural processing units could be applied for the identification of radar targets described by the set of separated scatterers, even in cases where the relative distance between the scatterers is comparable to or less than the effective width of each scatterer. The obtained results indicate a high accuracy estimation of separate scatterer centers in the presence of noise which is not limited to the stationary case but supposed to be cyclostationary. It was also shown that the parameters describing the coordinates of scattering centers could be successfully extracted from the trained ANN after about one hundred epochs spent on ANN training process, which is carried out by means of modified gradient

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descent method. The main result is to demonstrate the possibility of using neural networks to automatically analyze radar images, which is an integral part of a set of tasks that form the target recognition problem. The proposed algorithm implements an approach of identification systems made using a neural network training procedures.

Keywords*: scatterer estimation, non-stationarity, cyclostationarity, complex radar image, radar targets.*

АННОТАЦИЯ

В статье рассматривается проблема распознавания радиолокационных целей, выполняемая на сложных радиолокационных изображениях. Подход, основанный на искусственной нейронной сети (ANN) с радиальной базисной функцией (RBF), предлагается для идентификации точечных рассеивателей, размещенных в радиолокационном изображении. Обновленная концепция простых адаптивных блоков как основы для сборки сети позволяет разработать схему извлечения признаков на основе ANN для двумерной обработки сигналов. Было показано, что ANN, реализующий нейронные блоки обработки RBF, может применяться для идентификации радиолокационных целей, описываемых набором отдельных рассеивателей, даже в тех случаях, когда относительное расстояние между рассеивателями сопоставимо или меньше эффективной ширины каждого рассеивателя. Подход, представленный в этой статье, состоит в использовании нейронной сети с особо синтезированной радиальной базисной функцией (RBF), которая используется для аппроксимации отобранных радиолокационных изображений, передаваемых на ее вход. Полученные результаты указывают на высокую точность оценки отдельных центров рассеивателей при наличии шума, который не ограничивается стационарным случаем, а предполагается, что он является циклостационарным. Также было показано, что параметры, описывающие координаты центров рассеяния, могут быть успешно извлечены из обученного ANN после примерно ста эпох, потраченных на процесс обучения ANN, который осуществляется с помощью модифицированного метода градиентного спуска. Основным результатом является демонстрация возможности использования нейронных сетей для автоматического анализа радиолокационных изображений, что является неотъемлемой частью набора задач, формирующих проблему распознавания целей. Предложенный алгоритм реализует подход идентификации систем, выполненных с использованием процедур обучения нейронной сети.

Ключевые слова: *оценка рассеивателя, нестационарность, циклостационарность, комплексное радиолокационное изображение, радиолокационные цели.*

1. INTRODUCTION

One of the important problems solved by modern smart radar systems is target identification. The possible approach to its solution is the machine-based analysis of radar images which can be performed by means of automated scattering analysis of the radar targets. The conceptual idea here is a decomposing the whole image under processing into point scatterers (Rihaczek and Hershkowitz, 2000) which are rather simple elements whose appearance can have a clear physical explanation. It allows each scatterer to be reproduced as a part of mutually interconnected structure supported by the appropriate artificial neural network (ANN) (Efimov *et al.,* 2014; Sandu *et al.*, 2018).

The authors of the current paper found out (Efimov and Shevgunov, 2012b) that the existing methods of ANN design require some modifications in order to allow considering the neural networks as identification techniques rather than approximation models only. The essential benefit (Efimov and Shevgunov, 2012a) is that neural networks organized as identification models provide one with the powerful tool to perform effective parameter estimation procedure. The complex model can be decomposed into simpler blocks, which are mirrored in the appropriate units in the network graph (Dubrovin *et al.,* 2014; Koltunov *et al.*, 2018). It means that not only can a successfully trained network be used for representing the revealed dependency, but it can also extract the values of internal parameters featuring its elements. Having been extracted, these values can be reversely mapped into the values related the original model parameters. In many widespread cases (Shevgunov *et al.,* 2014), there would be a simple one-to-one reference between model parameters and some of network parameters.

A typical radar image to be processed is assumed to be obtained by a radar system performing azimuthal scanning with a high resolution in both distance and angle dimensions. The system operating in centimeter wavelength range (Shevgunov and Efimov, 2019a) emits coherent pulses using the same antenna, working with time division, both for the transmission and the echo measuring receive. The scanning cycle is inevitably carried out in the presence of noise that can be modeled as stationary or non-stationary, e.g. cyclostationary (Shevgunov, 2019; Shevgunov *et al.,* 2018a), process. The general concept of the scatterer identification approach is the estimation of parameters describing the set of scatterers involved in target's representation such as the coordinate of their centers within the target as the most important information to characterize its geometrical form in any further processing. This information being acquired could be passed to the customized classification systems, which can be a data-driven system organized on machine-learning principles that will form internal features working as anchors for the process of the automatic target identification.

The approach presented in this paper consists of using particularly synthesized radial basis function (RBF) neural network which is used for approximating sampled radar images given to its input. As soon as the approximation has finished successfully, the Cartesian coordinates of the scatterers are taken from the parameters of the neurons directly, which is proved by the intense numerical calculations whose results are presented in order to estimate the practical viability. It was shown in (Efimov and Shevgunov, 2014) that the ability of neural networks to operate on distorted, noisy and incomplete data sets alongside with the properties of RBF-neurons allows them to be applied in the scatter based target identification. The rest of this paper is organized as follows. Materials and Methods introduce the short outlook of the radar image model. The ANN-based solution for the problem identification of multiple scatterers and the numerical simulation are described in Results and Discussion. The paper ends with conclusion depicting the further development of the processing based on ANN.

2. MATERIALS AND METHODS

The basic model of a complex radar target (Henderson and Lewis, 1998) considers the radar target to be represented as a set of individual scatterers mounted on a stiff backbone. This model supposes that echo-response signals received during observation are to be determined as a superposition of the responses from each individual scatterer. Each of these individual responses possesses in the first consideration the form of the probe pulse emitted by the radar system. The physical model lying behind point scatterers is based on the assumption of the electromagnetic wave reflecting from the sharp edges of the man-made objects such as aircraft. Moreover, complex objects can be expressed as a unique superposition of scatterers as if they all were hold of a stiff frame. The model proposed has a few deliberately introduced simplifications. At first, non-linear distortions caused by signal reflection from a scatterer are thought to be compensated. At second, the changes in the positions of the scatterers are considered insignificant whatever they have been caused, i.e., due to antenna moving or micro vibration. Finally, the total inaccuracy of the reflection process could be represented in the model by means of the additive noise component.

The proposed model of space-time radar echo response signal of a complex radar target is used to generate test input radar image. Since a high resolution in both distance (denoted by ρ) and angle (denoted by φ) for the observed radar target is assumed, the signal could be described with Equation 1 where $\dot{x}_p(t,\theta)$ stands for complex-valued space-time radar signal echoed from p-th individual scatterer, $\dot{s}\left(t-\tau_p\right)$ is the slice of the radar image across the distance (in fact, the form of this slice will correspond to the form of probe pulse), $f_A^2\left(\theta-\varphi_p\right)$ is a term for squared antenna pattern, φ_p determines the main beam direction of the antenna. The term *n(t)* describes the additive noise which exhibits stationary or cyclostationary properties (Shevgunov *et al.,* 2018b; Shevgunov and Efimov, 2019b) in the frequency band width enough for efficient functioning of the radar system. The schematic structure of the reflected signal is shown in Figure 1, where one can see red thin solid line as the three-dimensional plot of the wave in the time domain. The amplitude response depends on the scanning angle due to the particular radiation pattern of the antenna as well as on the back-scattering radiation pattern of the scatterer under investigation. Since the latter is assumed to be a great deal wider than the former, one can conclude that the response envelope will resemble the squared radiation pattern of the antenna, which is depicted via thin blue solid line.

The example of a complex radar image possessing three individual scatterers generated according to the introduced model for the complex radar target is shown in Figure 2 where the intensity varies – the lighter pixels are, the greater values of the function they represent.

Nevertheless, one can bear in mind that the image is visualized only by the absolute value of the original complex-valued 2D function (1) and any information about the phase is omitted there. As one can note in Figure 2, two of three pulses are close to each other. Therefore their responses overlapping was chosen intentionally to investigate whether the proposed ANN algorithm is able to distinct them successfully and to what extent. The third pulse has the greatest intensity and located far separately from the others. The probe pulse range waveform and antenna cross-range pattern are both assumed to have shape of the Gaussian curves. The typical value of the width of the antenna cross-range pattern is 1–5 degree while the effective pulse width is from 1–10 ns which determines the temporal and spatial resolution for further processing.

3. RESULTS AND DISCUSSION:

3.1. Neural Network Design

Scattering center coordinates are usually considered (Chen and Andrews, 1980) to be the most relevant parameter for the target identification. Thus in (Konovaluk *et al*., 2010; Chen and Ling, 2002 the prolific way to the identification using parametric methods for pole estimation in the frequency domain authors has been proposed. The coordinates of the poles on the virtual complex plane can be used then to evaluate geometrical centers of the scatters. Although this approach demonstrates high accuracy and has proven suboptimal nature, it suffers from high calculation cost and the requirement to perform accurate deconvolution of the radar image that is a naturally ill-conditional problem. Authors of the current work have proposed in (Efimov and Shevgunov, 2013) alternative solution based on ANN-framework since taking into consideration the fact that Radial Basis Function (RBF) neural networks perfectly correspond to the model (1). The proposed approach contains the following steps:

Step 1. Radar image sampling.

Step 2. The preparation of the training set is made of the radar image samples.

Step 3. RBF neural network synthesis.

Step 4. Training of the ANN using the set as the input data.

Step 5. Extraction RBF neurons parameters are performed to directly calculate coordinates of individual scatterers.

The structural scheme of RBF artificial neural network is shown in Figure 3. It consists of input signals x and y representing coordinates of a point belonging to the image to be processed and the output z representing the intensity at this point; the block marked with “+1” introduces bias input. The training set consists of samples whose x and y coordinates in the image are used as input data, and the signal intensity in that point is used as target output data. Hence, the problem can be now overlooked as the task of approximation of the target radar image with neural network.

The output signal generated by ANN is defined by the Equation 2 where x_a is output network signal as the approximated radar image, ρ and φ stand for distance and antenna azimuthal angle correspondingly, P denotes the number of RBF neurons within the network, g_p is the partial output signal taken as weighted output of p-th RBF neuron. The key point to highlight is that single RBF neuron is targeting a particular part of the image which best corresponds to the neurons output signal. Therefore, by selecting activation function of the neuron in accordance to the probe pulse form and antenna pattern, one can expect that each single RBF neuron will target one individual scatterer provided the training process is successful. The parameters of the neurons are available after the training and can be directly used for calculating the estimated parameters of the scatterers, which are the coordinates of their centers and the effective widths.

The structure RBF-neuron is of particular interest as it is a special type of neuron that not only uses a radial basis function as its activation function but also has another input combiner. Structural scheme of RBF neuron during feed-forwarding is shown in Figure 4. In case of two-dimensional input data, the neuron will have 3 inputs: two are for coordinates x and y, and the extra one is for bias. The latter is marked with “+1”. The transformation function can be easily written from the scheme of the neuron in Figure 4. Thus, the output signal is defined by the Equation 3 where x_0 and y_0 are the coordinates of the centers, k_x and k_y are scale multipliers. The argument inside the parenthesis of the term f in the transfer function above can be expressed in the form of the canonical equation of an ellipse or an ellipsoid if the dimension would be greater than two (Equation 4) where the following substitutions are applied (Equation 5).

In the case of Gaussian function taken for the activation function f, the output signal of one RBF-neuron will be equal to unity at the central point and rapidly decreasing bell around. The individual knowledge of the adaptive elements behavior during their back-propagations opens the option for constructing a structural scheme of RBF neuron for back-propagation as it is seen in Figure 5.

3.2. Numerical Simulation

Since any strict theoretical investigation on the estimation ability of ANN is always challenging, the numerical simulation is a helpful tool that is used in order to estimate practical accuracy of the proposed procedure. In this section, the numerical simulation is being conducted according to the five-step plan developed in the previous section. In the first step, the described above allocation of three individual scatterers was used to synthesize the radar image of a complex target. Basically, the radar image previously shown in Figure 2 undergoes the sampling procedure, which is performed with the equal sampling steps in both dimensions. The discrete points, or samples, of the image are shown in Figure 6 by means of circles, which area is proportional to the intensity of the wave received at each point in the two-dimensional plot. Since the value is superposition of all the scatteres involved in imaging their values are not decaying smoothly from the center down to the side of the bell curved shown in Figure 2.

The second step consists in getting all the samples together in order to form the training set as the set of ordered pairs *{(x, y), z}*, where *x* and *y* are spatially coordinated of each sample and *z* is the absolute value of the intensity of the reflected field at the point with the given position. In the third step, there is synthesized the ANN based on RBF architecture described in the previous section. Since the number of RBF-neuron corresponds to the number of scatterers, this quantity is assumed to be known and equal to three. The activation function is chosen Gaussian in accordance with the form of probe pulse and antenna pattern possessed by model (1). The assumption of the Gaussian bell is not critical for the overall performance as it will be seen it further.

In the fourth step, the training set containing all the samples was used to train the RBF ANN network in the batch mode with the gradient descent method. The choice of the gradient descent method was made due to its relative simplicity even at the cost of slower convergence in comparison with the second-order method. However, the latter are required computationally expensive procedures such as the matrix inversion, which is the gradient descent method is free of. For the purpose of controlled training process, the objective function was defined as the mean-squared error (MSE) function, which evaluates the difference between the source radar image and the solution evaluated at particular point by the synthesized RBF ANN. In addition, it evaluates the overall network performance while the training goes on and gets the clue about the epoch number when the training process can be stopped. The learning curve during the training is shown in Figure 7 as the value of the MSE curve plotted against the number of iteration or epoch.

The curve in Figure 7 can be split into two parts. In the first part, the error is large but decreases rapidly while, in the second part, the error is rather small but goes down slowly. One can see that the low level of MSE is achieved just after 100 training iterations; hence, the approximated radar image fits the source radar image accurately to process further. Besides, Figure 7 shows that the value of MSE is still decreasing at the end of the training process but notably slower than it took place at the beginning.

In the last step, the parameters of the scatterers are immediately extracted from the adaptive elements which the neurons consist of. Since the training process has finished successfully, the neurons contain the values related to the positions of the scatterers. The reconstructed scatterers are shown in Figure 8 alongside with the true ones. The black solid lines depict the two-sigma level of each scatterer in the model whereas the dashed blue line illustrates the same for the scatterers estimated by the RBF ANN. It is clear that the centers of all scatterers are accurately estimated despite the fact that two of them were chosen in the model to be partially overlapping. However, the effective widths of the scatterers are not estimated so accurately. The width of two of the scatterers was estimated smaller as one can see in Figure 8 that the blue ellipses are inside the black ones. This phenomenon could be explained by their lesser influence on the overall MSE value and will be discussed in the next section in more details.

4. CONCLUSIONS:

The series of Monte-Carlo simulation clearly indicates that RBF neural network can be successfully used for accurate estimating the centers of the individual scatterers with signal-to-noise ratios down to 5 dB. However, the accuracy of the parameter describing the effective width of each scatterer is rougher. The explanation of this phenomenon consists in the difference in the influence caused by parameters of both types on the overall target function to be minimized during ANN training prosses. Thus, that function is significantly more sensitive to the change of the position of the scatter due to the fact that the greater values of the reconstructed image are primarily concentrated in the neighborhood of their center. Since the target function is quadratic, the greater values serve as additional weights. In contrast, the estimated values of scatterer width may vary in the wider range without any significant change in the target function that can be enough to adjust them even in the later training epochs. Nevertheless, the positions of the scatterers are considered to be more robust features as they are strongly related to the physical geometry of the target while the intensity of the reflection, which determines the width, will vary depending on many factors, the most important of which is the angle of the incident electromagnetic wave.

The paper reflects the current advances in the ANN based signal processing in regards to tasks related to the modern signal processing systems. The main result of the paper is demonstrating a possible way how neural networks can be used for automated radar image analysis which is the essential part of a set of tasks forming target recognition problem. The proposed algorithm carries out the system identification approach reached via neural network learning procedure. Thus. the radar image is firstly approximated by RBF networks where each RBF-neuron preserves the information about the point scatterer of the possible targets. The adaptive element concept chosen for ANN synthesis is extremely suitable for the second stage when the values of parameters are being extracted and further transformed into parameters of the multiscatterer model. The research, whose some results are presented in the paper, is bound to be continued as it is expected to be naturally developed into a universal scatter estimator that can be used for the automated scatterer identification and further reconstruction of the target shape carried out by means of another ANN belonging to the class of multilayer perceptrons.

5. ACKNOWLEDGMENTS:

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$$\dot{x}(t,\theta) = \sum_{p=1}^{P} \dot{x}_p(t,\theta) + n(t) = \sum_{p=1}^{P} \dot{a}_p \cdot \dot{s}\left(t-\tau_p\right) \cdot f_A^2\left(\theta-\varphi_p\right) + n(t) \quad \text{(Eq. 1)}$$

$$x_a(\rho,\varphi) = \sum_{p=1}^{P} g_p\,(\rho,\varphi) \quad \text{(Eq. 2)}$$

$$z = f\left(k_x(x_0+x)^2 + k_y(y_0+y)^2\right) \quad \text{(Eq. 3)}$$

$$z = f\left(w(x,y)\right),\ \frac{(x-x_1)^2}{c^2} + \frac{(y-y_1)^2}{d^2} = 1. \quad \text{(Eq. 4)}$$

$$x_1 = -x_0, y_1 = -y_0,\ c = \sqrt{w/k_x}, d = \sqrt{w/k_y} \quad \text{(Eq. 5)}$$

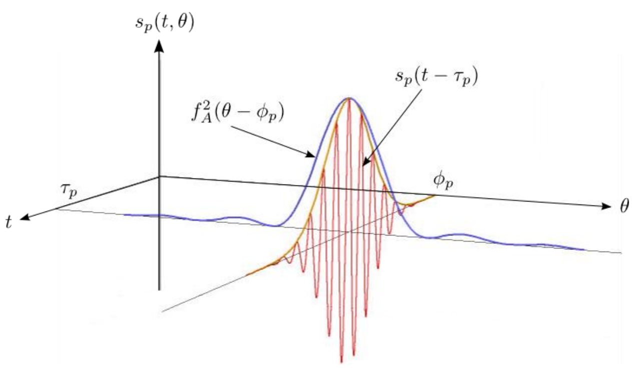

Figure 1. *The structure of space-time radar echo response signal of complex radar target*

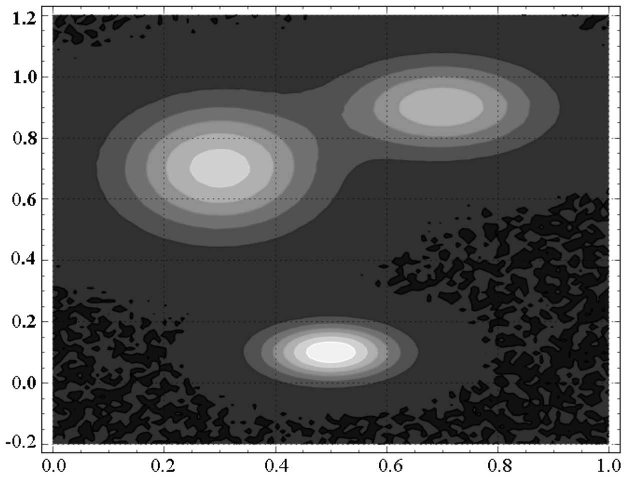

Figure 2. *The example of a radar image with three scatterers*

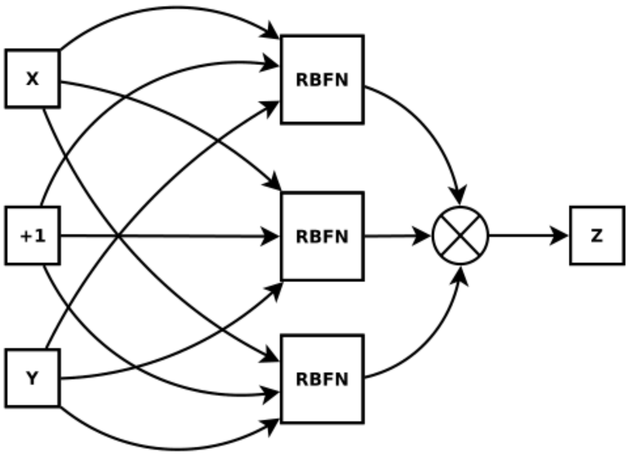

Figure 3. *RBF neural network*

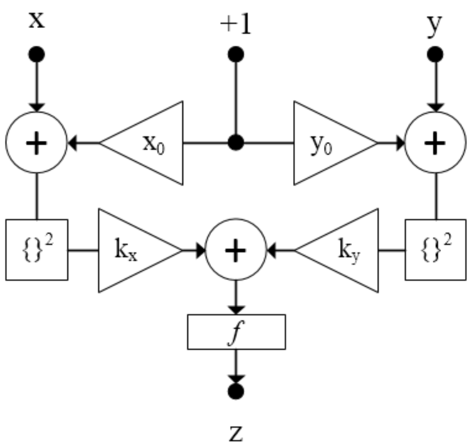

Figure 4. *The scheme of RBF neuron for feed-forwarding*

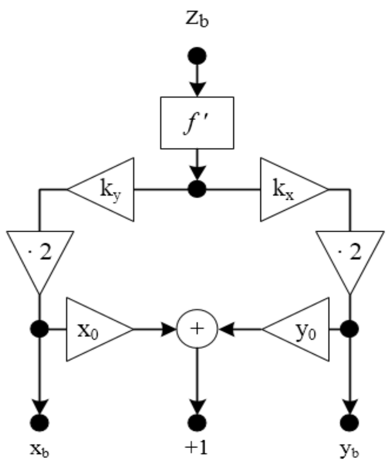

Figure 5. *Structural scheme of RBF neuron for backpropagation*

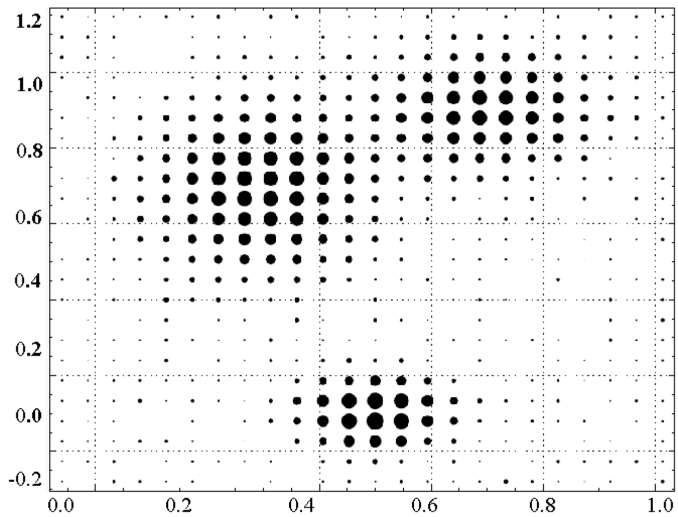

Figure 6. *Sampled radar image*

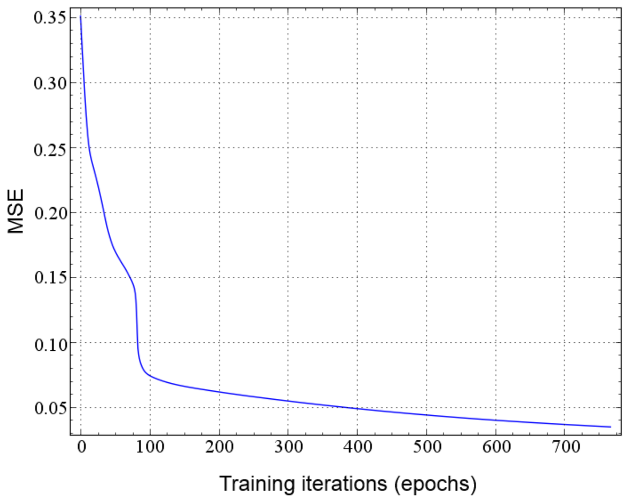

Figure 7. *Training process*

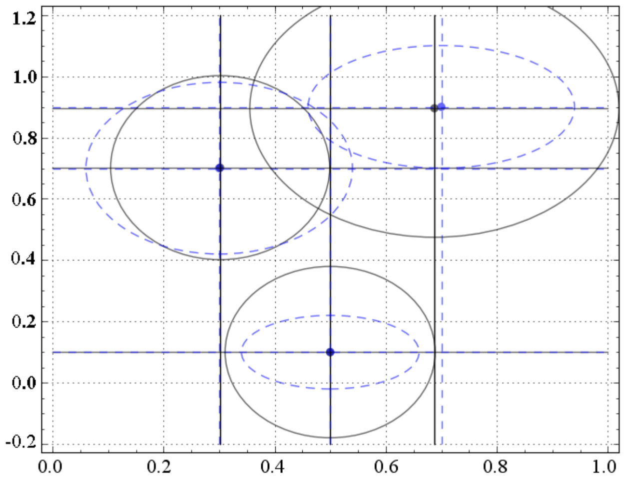

Figure 8. *Original and reconstructed scatterers*