PERIÓDICO TCHÊ QUÍMICA
ARTIGO ORIGINAL

ANÁLISE DOS ERROS DE CONEXÃO MATEMÁTICA DOS ALUNOS NA RESOLUÇÃO DE PROBLEMAS DE IDENTIDADE TRIGONOMÉTRICA

ANALYSIS OF STUDENTS' MATHEMATICAL CONNECTION ERRORS IN TRIGONOMETRIC IDENTITY PROBLEM SOLVING

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RESUMO

A identidade trigonométrica é importante no aprendizado de matemática, porque exige que os alunos pensem crítica, lógica, sistematicamente e completamente. A resolução de problemas de identidade trigonométrica exige que os alunos relacionem conhecimento conceitual ou conhecimento processual, que depois é usado em perguntas. Este estudo envolveu alunos da série X do ensino médio, que foram examinados para descobrir os tipos de erros de conexão matemática e as causas dos erros. Antes das entrevistas baseadas em tarefas, 36 estudantes foram inicialmente submetidos a um teste. Com base em várias considerações, sete estudantes (três homens e quatro mulheres) foram selecionados para serem submetidos a uma entrevista baseada em tarefas. Esta pesquisa empregou um método qualitativo de pesquisa com desenho de estudo de caso. Os resultados da pesquisa indicam que os erros na conexão com o conhecimento conceitual são mais comumente o erro de conectar o conceito algébrico. Por outro lado, 86,11% dos estudantes experimentaram erros na conexão com o conhecimento processual. Esse erro ocorreu quando os alunos trabalharam em problemas com identidades trigonométricas que raramente encontravam em exercícios. Erros nas conexões matemáticas da identidade trigonométrica são causados pela falta de compreensão da operação aritmética algébrica, ênfase no conceito e conhecimento estratégico. Isso mostra que os alunos precisam de uma variedade de problemas para poderem dominar várias formas de identidades trigonométricas. O resultado desta pesquisa também reforça o importante papel dos conceitos algébricos como conhecimento prévio no estudo da identidade trigonométrica.

Palavras-chave: conexão matemática; erro; conhecimento prévio; processual; conceptual

ABSTRACT

The trigonometric identity is essential in learning Mathematics because it requires students to think critically, logically, systematically, and thoroughly. Solving trigonometric identity problems requires students to relate conceptual knowledge or procedural knowledge, which then used in questions. This study involved grade X students of senior high school, which were examined to find out the types of mathematical connections errors and causes of the errors. Before task-based interviews were conducted, 36 students were first given a test. Based on several considerations, seven students (three males and four females) were selected to undergo a task-based interview. This research employed a qualitative research method with a case study design. The results of the analysis indicate that the errors in connecting to conceptual knowledge are most commonly the mistake of connecting the algebraic concept. On the other hand, 86.11% of students experienced errors in connecting to procedural knowledge. This error happened when the students worked on problems with trigonometric identities, which they had rarely encountered in exercises. Errors in mathematical connections in trigonometric identity are caused by the lack of understanding of the algebraic arithmetic operation, emphasis on the concept, and strategic knowledge. It shows that students need a variety of problems to be able to master various forms of trigonometric identities. This research's result also reinforces the critical role of algebraic concepts as prior knowledge in studying trigonometric identity.

Keywords: mathematical connection; error; prior knowledge; procedural; conceptual

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1. INTRODUCTION:

Trigonometry is one of the earliest mathematics topics that link algebraic, geometric, and graphical reasoning; it can serve as an essential precursor towards understanding pre-calculus and calculus (Weber, 2005). Trigonometric identities play a significant role when students study geometry and calculus at the next level. The essence of learning trigonometric identity is not only knowing its knowledge but also understanding the discovery or derivation of its properties. The trigonometric identities hone students' ability to think deductively, creatively, and practice the problem-solving skill.

During the process of learning mathematics, students need the ability to relate various mathematical concepts as conveyed in Bruner's connectivity theorem, "in mathematics, each concept relates to other concepts." Therefore, students must be allowed to make mathematical connections to be more successful in learning mathematics. Mathematical connections are explained by Hiebert and Carpenter (1992) as part of structured networks such as spider webs, "The junctures, or nodes, can be thought of as pieces of represented information, and threads between them as the connections or relationships." Hiebert and Carpenter attributed the process of conceptualizing the conceptual knowledge to the corresponding structure, which was then highlighted that knowledge only exists when some insight can be interconnected. Also, conceptual understanding can be formed by increasing the number of connections between these pieces of knowledge.

Mathematical connections relate with the connection between one concept to another, one procedure to another, as well as the relationship between facts and procedures, we can see that mathematical connection are related to the ability to link conceptual knowledge and procedural knowledge. Students need both applications of theoretical and procedural knowledge to solve any problem correctly (Taconis, et al., 2000; Cracolice, et al., 2008). Thus, we can divide the mathematical connection errors into two types, namely errors in connecting to conceptual knowledge and errors in connecting to procedural knowledge (Table 1).

Procedural knowledge is a knowledge that focuses on skills and step-by-step procedures without explicit reference to mathematical ideas (Marchionda, 2006). Meanwhile, conceptual knowledge, according to McCormick (1997), relates to the relationship between various knowledge, so that when students can identify this relationship, it can be said that they have a conceptual understanding. Surf's research (2012) showed that there is a significant relationship between theoretical knowledge and procedural knowledge in solving problems.

The study aimed to find out the types of students' mathematical connections errors and causes of the errors. Insights provided by this research can serve some useful indications such as an error in connecting to conceptual or procedural knowledge, which was done by students and the one that became our particular concern. Besides, it is expected to be useful as an input for teachers about the importance of making mathematical connections for students so that they can be considered in teachers' plans and learning instruction.

2. LITERATURE REVIEW:

Many studies have concerned about misconceptions and making errors (Bush, 2011; Dhlamini & Kibirige, 2014; Sarwadi & Shahrill, 2014; Mohyuddin & Khalil, 2016; Ndemo & Ndemo, 2018; Sudihartinih & Purniati, 2020). A few researchers have also mentioned students' misconceptions and errors in trigonometry, such as Orhun (2015) and Usman & Hussaini (2017). Usman & Hussaini's research is based on Newman Error Hierarchical Model, which differs from this research in terms of research method, which was used. Orhun's research is about the difficulties faced by students in using trigonometry for solving problems in trigonometry. Although Orhun studied the students' ability to develop the existing concept in trigonometry, the focus of his research is the trigonometric ratio. On the other hand, Brown (2006) studied students' understanding of sine and cosine. He found out a framework, called trigonometric connection. Although Brown's research was on trigonometric link, the focus of the study was only on the rules of sine and cosine; therefore, this research focuses on trigonometric identity. The identity of trigonometry is one of the topics that high school students should study. Orhun (2015) found that the students did not develop the concept of trigonometry with certainty.

Various literature (NCTM, 2000; Mousley, 2004; Blum, et al., 2007) have identified two main types of mathematical connections. The first is to recognize and apply mathematics to contexts outside mathematics (the relationship between mathematics, other disciplines, or the real world). The second concerns the interconnections between ideas in mathematics.

Moreover, students can understand mathematics deeply if they have an understanding of conceptual and procedural knowledge (Wilkins, 2000). Conceptual knowledge focuses on understanding the relationship between mathematical ideas and concepts. Meanwhile, procedural knowledge focuses on symbolism, skills, rules, and algorithms that used step by step in completing mathematical tasks. Students must learn the concepts at once with procedures so that they can see the relationship. In line with Wilkins, the NCTM's Principles and Standards for School Mathematics (2000, p. 35) states that developing fluency in mathematical problem solving requires a balance and connection between conceptual understanding and computational ability. The procedures used by the students while solving mathematical problems show the various levels of students' conceptual understanding (Ghazali & Zakaria, 2011). Meanwhile, the issue of conceptual knowledge is a problem experienced even by undergraduate students and prospective teachers (Arjudin, et al., 2016; Dündar & Gündüz , 2017).

3. MATERIALS AND METHODS:

3.1. Research methods and participants

Qualitative research method with case study design was used to investigate the grade X students in a high school located in Kartasura, Indonesia. The reason for choosing this school was because the percentage of national examination results on high school trigonometric from 2013-2017 was lower than another topic. Moreover, the results of interviews with teachers indicated that students have difficulty in studying the identity of trigonometry than other issues. Also, the researchers did not teach in the school; it is to minimize the bias of research, which may occur due to the subjectivity of the researchers with students and school staff.

Researchers have received approval to research the school concerned through a letter of agreement signed by the Assistant Principal of Public Relations and Partnership. Moreover, all participants have agreed to participate in this study. Researchers kept their identity to be confidential.

The qualitative approach was appropriate because the analysis of the learners' responses to the given was used to generate theory (Dhlamini & Kibirige, 2014). The case study method was chosen in the hope that through the study of a somewhat unique individual, valuable insights would be gained (Fraenkel & Wallen, 2000).

3.2. Data collection

This research used task-based interviews that have been used by researchers in qualitative research in mathematics education to gain knowledge about an individual or group of students' existing and developing mathematical knowledge and problem-solving behaviors (Lerman, 2014). The tasks used in this research consists of three items with various degrees of difficulty. It is intended to identify the mathematical connection errors of upper students and anticipate the possibility that only a few students who can answer the questions. Problems given in this research are as follows:

Problem A (for the first test):

1) Prove that $sec\,x - sec\,x sin^2 x = cos\,x$!
2) Known:
   - $a\,cos\,x + tan\,x = sec\,x$ and
   - $-\,b - tan\,x = sec\,x,$
   - determine the simplest form of a.b !
3) Prove that $\frac{cos\,x}{1 + sin\,x} = \frac{1 - sin\,x}{cos\,x}$!

Problem B (for the interview):

1) Prove that $csc\,x - csc\,x\,cos^2 x = sin\,x$!
2) Known:
   - $a\,sin\,x + ctg\,x = csc\,x$ and
   - $-\,b - ctg\,x = csc\,x,$
   - determine the simplest form of a.b !
3) Prove that $\frac{sin\,x}{1 + cos\,x} = \frac{1 - cos\,x}{sin\,x}$!

Question number one is a matter of easy category since it only tests students' ability to relate their knowledge of distributive properties (a matter of pre-condition) and then comparing it to the concept of trigonometric identity. Problem number two is a tricky category because it examines the ability of students to identify the relationship between the first equation and the second equation, which can be operated to produce a trigonometric identity formula. So, this problem requires students' ability to relate the concept of arithmetic algebra and then linking the results obtained to the idea of trigonometric identity. Problem number three is a matter of medium category because it tests the ability of students in doing algebraic manipulation that can generate trigonometric identity form associated with the form of trigonometric identity, which will be proven.

Before task-based interviews are conducted, 36 students were given a test first (see Problem A). Task-based interviews are conducted for students who have different content analysis results (different from other students) and fulfill almost every indicator, based on these considerations, seven students (three males and four females) were selected to undergo a task-based interview. Task-based interviews are carried out to get valid data about errors in mathematical connections that students do and to get data about the causes of failures. Accurate data regarding the purpose of the failure is obtained by conducting interviews at different times.

During the task-based interview, the researchers gave the test to the students in which the problem of the test was similar to the question presented in the previous test (see problem B). The problem is made similar to avoiding the possibility of students memorize the answers. The data were in the form of statements about things done during the fulfillment of tests, errors in making a mathematical connection, causes of mistakes, and guidance from the researcher.

3.3. Data analysis

The study used a content analysis technique that was also used by Luneta (2015) when researching students' misconceptions. In this technique, each question is analyzed according to the content they contain (student errors); in this case, the students' answers are indicative of their ability to interact with trigonometric identity questions. The variables measured are their responses (misconceptions and related errors) to the correct answers. The analysis made inferences to the communication (student's answers) by systematically and objectively identifying specific characteristics of the student's errors in the answers. The unit of analysis was the errors students displayed on each questions. Errors made by subjects are grouped and identified its characteristics. Indicators of the type of mathematical connection errors can be expressed in Table 1.

4. RESULTS AND DISCUSSION:

The following is the analysis of the mathematical connection error that students do in working on the problem of trigonometric identity along with its causes. A summary of students' errors on the items is provided in Table 2.

In the transcript, the researcher is referred to as R meanwhile, the seven students interviewed are referred to as S.N.2, S.N.5, S.N.6, S.N.12, S.N.1, S.N.33, and S.N.34 which imply the students' serial number in the class. There is also an abbreviation in the figure such as K.S.17.1. K.S refers to student errors, 17 refers to the serial number of students in the class, and 1 refers to the error number.

4.1. Error in mathematical connection problem number 1

4.1.1 Type A

In question number 1, four students include subject number 33, failed in relating knowledge of algebraic arithmetical operations. During the initial test, the student performed a reduction operation on sec x – sec x $\sin^2 x$, which resulted in $\sin^2 x$. When the student was asked to solve a question for a task-based interview, the student redid the same mistake. It can be seen from the quotation I as follows:

Quote I

S.N.33: $\frac{1}{\sin x} - \frac{1}{\sin x}\,(1-\sin^2 x)$ results in (1-$\sin^2 x$) because $\frac{1}{\sin x}$ - $\frac{1}{\sin x}$ the result is zero.

R: Look, $\frac{1}{\sin x}$ multiplied by1- $\sin^2 x$, right?

S.N.33: Yeah ...

R: Let this *a* (the researcher designated $\frac{1}{\sin x}$) minus *b* (the researcher pointed 1-$\sin^2 x$) multiplied by *a*, what is the result?

S.N.33: *a – ab*

R: So, is it true if you write it as this (designate $\frac{1}{\sin x} - \frac{1}{\sin x}\,(1-\sin^2 x)$)? Is it true if the result is 1-$\sin^2 x$?

S.N.33: No ...

According to quote I, the student thought that $\frac{1}{\sin x}$ - $\frac{1}{\sin x}$(1- $\sin^2 x$) resulted in 1- $\sin^2 x$. It shows that the student failed in relating the concept of algebraic form calculation, which had been learned in junior high school to be applied to this problem. That algebraic form contains fractions, Bush (2011) in his research, showed that students often applied the wrong algorithm when adding, subtracting, multiplying, or dividing fractions. When the researcher simplified the algebraic form, the student was able to answer the researcher's question correctly. So, this error occurred since the student did not understand the concept of algebraic arithmetic operations. Problems regarding understanding algebraic concepts are problems that have attracted the attention of many researchers (House & Telese, 2008; Nathan & Koellner, 2007; Bush, 2011; Pournara, et al., 2016), this suggests that the notion of algebraic concepts are things that are commonly experienced by students. The results of this research, which focuses on trigonometric identities, also found students who lacked understanding of algebraic concepts; this finding supports the research of Usman & Hussaini (2017) which focused on the manipulation of trigonometrical ratios using formula and the right-angled triangle. They noted that the students' error in solving trigonometrical problems was due to their weaknesses in basic arithmetical operations.

4.1.2 Type B

In question number 1, four students failed in relating their knowledge about distributive properties to change an algebraic form. One of four students who experienced this failure is subject 33, as seen in the following quotation II:

Quote II

R: In this "$a - ab$" form (a stands for $\frac{1}{\sin x}$ and b stands for (1-sin^2x)), can you change its shape based on its distributive properties? (Students were asked to change "$a - ab$" form into "$a(1 - b)$" form)

S.N.33: Yes ..... it becomes $a - b$.

R: Really? If we apply distributive properties to this form, it will become "a". Then, what is inside the bracket?

S.N.33: Uh, it becomes 1.

R: Okay, 1 and then?

S.N.33: Minus b.

The student cannot explore her cognitive ability about distributive properties on $\frac{1}{\sin x} - \frac{1}{\sin x}\,(1 - \sin^2 x)$ becomes $\frac{1}{\sin x}$ (1- (1-sin^2x)) or becomes $\frac{1}{\sin x} - \frac{1}{\sin x} + \frac{1}{\sin x}.\sin^2 x$. It indicates that the student has failed to relate her prior knowledge. Gentile & Lalley (2003) revealed that the mastery of prior knowledge was required by students in the study of mathematics, it is an important step in concept development. This happens because the learning process in Mathematics is categorized as a widely related hierarchical learning process (Usman & Hussaini, 2017). If prior knowledge or skills do not exist, the task of learning becomes more difficult, if the prior knowledge has been wrongly understood, the need to learn a new topic results in students for getting previous misconceptions.

Based on quote I, students still encounter difficulties and need guidance from the researcher to apply distributive properties although $\frac{1}{\sin x} - \frac{1}{\sin x}\,(1 - \sin^2 x)$ has been simplified its algebraic form into a-ab. Thus, we can conclude that the cause of these errors is the inability to apply their basic knowledge of distributive properties to new situations. Mulungye,*et al*. (2016) stated that the distributive property says that $a(b + c) = ab + ac$. Therefore, students use this rule in new situations that are not appropriate. The distributive properties of the multiplication toward the sums are memorized in their minds that they intuitively misapplied them to a similar situation.

Students also failed to relate their knowledge about 1- sin^2x = cos^2x, which was then applied to question number 1. The four students use similar methods in solving problems. They changed $\sec x$ to $\frac{1}{\cos x}$, during the first test, but all four students made a mistake when changing the form of $\csc x$ in the second test. The students did not know that another form of $\csc x$ is $\frac{1}{\sin x}$ although the researcher had tried to guide them; so, they still cannot answer the question correctly. The fact that they can change $\sec x$ to $\frac{1}{\cos x}$, but cannot change $\csc x$ to $\frac{1}{\sin x}$ Indicates that they memorize the similarities that exist in trigonometric identities. Tobias (1993) stated that mathematical anxiety can cause a person to forget. This is consistent with the results of further interviews; the subject number 34 said that he felt depressed and considered the topic confusing, this anxiety caused him to forget the trigonometry identities. Mathematical anxiety is a feeling of tension and anxiety that interferes the manipulation of numbers and solving mathematical problems in various academic situations and everyday life. Some students tend to experience this anxiety in exams. Meanwhile, others are only afraid of mathematical calculations. Wells (1994) identified anxiety as a significant factor that hinders students' reasoning, memory, understanding of general concepts, and respect to Mathematics.

4.2. Error in mathematical connection problem number 2

4.2.1 Type A

As many as 44.4% of students failed to relate their knowledge about the concept of algebraic operation, especially in trigonometric identities. Students concluded that if 1- sin$^2 x$ = cos$^2 x$ then 1-sin x = cos x. The student thought that $\sqrt{1} = \sqrt{\sin^2 x + \cos^2 x}$ produces 1 = sin x + cos x. Students considered that the square root function was linear. Students tend to over-regenerate what has been experienced as something 'true' (De Bock, Van Dooren, Janssens, & Verschaffel, 2002)

The failure to relate knowledge of algebraic arithmetic operations also occurred when changing the form of a cos x + tan x = sec x into a = -cos x - tan x + sec x. This error occurred since the student did not understand the concept of addition, reduction, and multiplication, which are operated together. The researcher found that when the algebraic form was still in a complicated form, the students were not able to cope, but when the algebraic form was already in a simple form, the students were able to overcome it. Researchers found that students can overcome errors in connecting the concept of algebraic operations when researchers simplify the algebraic form. The student was able to answer the question and followed the direction given by the researcher when the researcher simplified the algebraic form and gave direction to him. Thus, the cause of this error was that the students do not understand the concept of algebraic operation. The findings of this research support Norasiah's research (2002) which noted that most average students faced difficulty in performing trigonometrical operations.

4.2.2 Type B

As many as 52.78% of students did not try to associate trigonometric identities with other trigonometric identities. Still, they chose to elaborate trigonometric identity formulas, which lead to more complex counting operations. If the student was able to handle such complexity as student number 26, he would achieve the simplest form. If the student was unable to handle it, he would have difficulty in finding the most straightforward way; it is experienced by subject number 17 (Figure 1).

When the students were asked to work on similar problems and then interviewed about the results of their work, they reused this strategy. It can be seen in the following quotation III

Quote III

R: Why did you choose this strategy? Did you elaborate each of these identities?

S.N.17: If it was not elaborated, the answer would not be the simplest one ...

R: Oohh… like that … do you know the identity which has something to do with csc *x* and ctg *x*? Something with a quadratic form?

S.N.17: Yes ...

R: What is it?

S.N.17: Cosecant *x* is equal to one per sinus *x*

R: No …something about csc^2*x*…

S.N.17: csc^2*x* = $\frac{1}{\sin^2 x}$

R: No…cosecant *x* is connected with cotangent *x*…

S.N.17: I forgot …

R: Do you ever think about this? This is ctg *x*, and this is csc *x*; then, its *b* is also there ctg *x*, csc *x* … then you look for *a* itself…*b* itself which were then sought the relationship between the two?

S.N.17: No …It is hard to think.

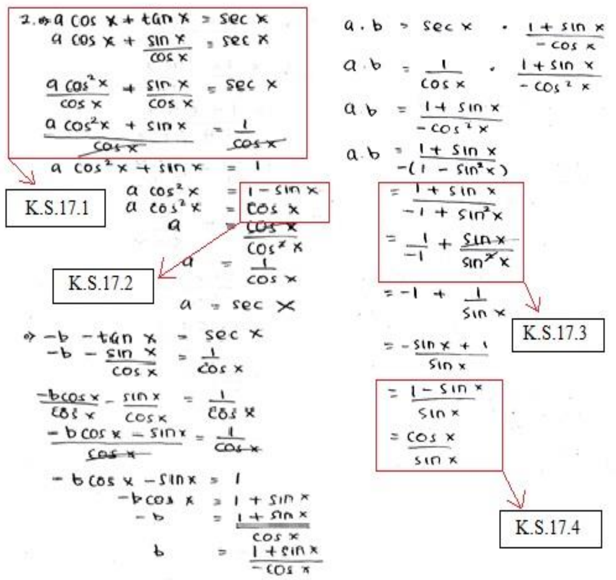

Figure 1. *The response of subject number 17*

Based on the quotation III, it indicates that the student failed to associate his knowledge about sec$^2 x$ – tan$^2 x$ = 1, which was then applied to the results of multiplication *a* and *b*. It happened because the student thought that a simple form would be achieved if each trigonometric identity was altered. The selection of such strategy resulted in complicated calculations; in this case, student number 17 was not able to overcome the complexity, which resulted in a miscalculation (see K.S.17.3 in Figure 1). Wright (2014) stated that students need the ability to choose the applicable formula based on the context of the mathematical problems they encounter.

As many as 5.6% of students chose the strategy of eliminating the system of equations, which caused some more complex counting operations. Although the students tried to solve this problem by linking their knowledge of elimination rules, this strategy is not appropriate. It indicates that the students did not understand the concept that the two equations are not linear equations (error type A). Although the students could change the equations in the form of linear equations, the number of unknown factors was more than the sum of the equations. It caused the system of linear equations not to have a solution. If it had a solution, it would have an infinite number of solutions. Besides, the student also failed to associate the fact that $\sec^2 x - \tan^2 x$ is equal to 1 (error type B). One of the students using this strategy was student number 6, as seen in the following quotation IV:

Quote IV

R: Why did not you try another strategy to solve this problem? Such as determining the value of *a* and *b* first, then doing the multiplication operation towards *a* and *b*?

S.N.6: Uumm …I did not think like that.

R: Was it because there were two equations, then you chose to eliminate them?

S.N.6: Yes… if there are two equations, I solve it by elimination.

Based on quote IV, it shows that the student eliminated both equations (see K.S.6.1 and K.S.6.2 in Figure 2). The cause was the student's lack of understanding of the concept of the linear equation system. The student assumed that two equations had to be solved by elimination. The choice of such a strategy resulted in more complicated calculations. In this case, subject number 6 was not able to overcome the complexity which occurred (see K.S.6.3 and K.S.6.4 in Figure 2) due to complicated forms, resulting in the student making errors in the calculations.

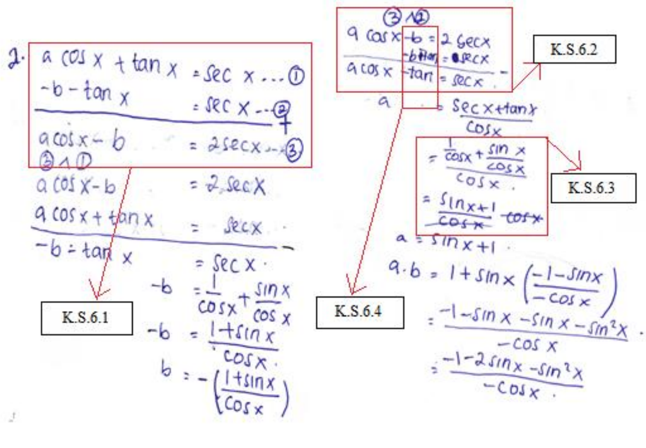

Figure 2. *The response of subject number 6*

Based on the results of the analysis on students number 6 and 17, it shows that the students did not have good ability to relate one trigonometric identity with other trigonometric identities. Based on the interview, all students who made mistakes in number 3 did not know the identity value of the $\sec^2 x - \tan^2 x$, even though, based on observations, the teacher has given detailed knowledge about $\sec^2 x - \tan^2 x = 1$. However, the teacher did not provide enough emphasis on this identity. The given problem was less varied. It only focused on the relationship of $\sin^2 x + \cos^2 x = 1$. In this case, the researcher assumed that the cause of the student's error in relating his knowledge of $\sec^2 x - \tan^2 x = 1$ to be used in the results of multiplication *a* and *b* was the lack of emphasis on the concept and lacking of strategic knowledge. This is in accordance with the results of Lopez's (1996) study which focuses on mathematical word problems, he also found that weakness in understanding concepts and lacking of strategic knowledge result in difficulties in problem-solving. Moreover, students who were weak in conceptual understanding were found to lack arithmetic and procedural skills (Narayanan, 2007).

4.3. Error in mathematical connection problem number 3

4.3.1 Type B

Some students failed to relate their knowledge of multiplication to convert 1 + cos *x* into1-cos^2*x*. Based on the interview result, it shows that the student already knew that he had to use the fact that 1-cos^2*x* is equal to sin^2*x* to solve this problem. However, the student could not exploit his cognitive ability to change 1+cos*x* into 1-cos^2*x*. It indicates that the student failed in relating his knowledge of multiplication with a conjugate pair. The student already knew which identity formula should be utilized after getting directions from the researcher. However, the student did not know how to direct 1 + cos *x* to be converted into 1-cos^2*x*. So, the cause of student's error was less skill in doing the algebraic manipulation.

The error which occurred in problem number 3 is classified as minimal since only seven students who experienced this problem. Moreover, no student experienced error type B on this problem. Based on the observation done by the researcher, the teacher has given practice and provided a discussion on this kind of problem. Besides, some problems with the students' worksheets also honed their ability to solve this kind of problem. If the students are accustomed to solving certain types of problems, they will be able to solve other problems that are similar in types; it is because the students already understand the way of settlement. A question or a mathematical problem is said to be a problem if the person who faces the challenge feels the gap between where it is and where it should be but do not know how to cross the deficit, this results in difficulties in solving existing problems (Reid & Yang, 2002).

5. CONCLUSION:

The results of this study have provided a new description of the error of mathematical connections in solving trigonometric identities. There are two types of mathematical connection errors. The first one is errors in connecting to conceptual knowledge (type A), and the second one is errors in connecting to procedural knowledge (type B). Students most commonly do error type A is the mistake of connecting the algebraic concept. Although the percentage is small, we can see that in-depth conceptual understanding of the linear equation system can cause connection errors in trigonometric identity problem solving.

In the second type (type B), an error occurred most often when the students worked on problems with trigonometric identities, which they had rarely encountered in exercises; 86.11% of students experienced this error. If the given problem contained common trigonometric identities encountered in their daily practice, only 30.56% students experience connection errors, from this fact, we could include that the second type of error was caused by the lack of emphasis on the concept and lack of strategic knowledge. Errors committed by the students in learning trigonometry can be useful for the teachers in evaluating their teaching as well as being able to correct the students' works appropriately.

6. LIMITATION AND STUDY FORWARD:

In this paper, we just discussed the error of mathematical connections in solving trigonometric identities. For the next researcher can discuss the topics more widely.

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8. REFERENCES:

1. Arjudin, Sutawidjaja, A., Irawan, E. B. & Sa'dijah, C. (2016). Characterization of Mathematical Connection Errors in Derivative Problem Solving. *IOSR Journal of Research & Method in Education,* 6, 5, 7-12.
2. Blum, W., Galbraith, P. L., Henn, H.-W. & Niss, M. (2007). *Modelling and applications in mathematics education.* New York: Springer US.
3. Brown, A. S. (2006). *The trigonometric connection: students" understanding of sine and cosine.* Prague, PME30, 228.
4. Bush, S. B. (2011). *Analyzing common algebra-related misconceptions and errors of middle school students,* University of Louisville: Unpublished Dissertations.
5. Cracolice, M. S., Deming, J. C. & Ehlert, B. (2008). Concept learning versus problem solving: a cognitive difference. *Journal of Chemical Education,* 85, 6, 873-878.
6. De Bock, D., Van Dooren, W., Janssens, D. & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of Secondary School Students' Errors. *Educational Studies in Mathematics,* 50, 3, 311–334.
7. Dhlamini, Z. B. & Kibirige, I. (2014). Grade 9 Learners' Errors And Misconceptions In Addition Of Fractions. *Mediterranean Journal of Social Sciences,* 5, 8, 236-244.
8. Dündar, S. & Gündüz , N. (2017). Justification for the Subject of Congruence and Similarity in the Context of Daily Life and Conceptual Knowledge. *Journal on Mathematics Education,* 8, 1, 35-54.
9. Fraenkel, J. R. & Wallen, N. E. (2000). *How to Design & Evaluate Research in Education.* Boston: McGraw-Hill Companies, Inc.
10. Gentile J. R & Lalley, J. P. (2003). *Standards and Mastery Learning Aligning Teaching and Assessment so All Children can Learn.* Thousand Oaks: Corwin Pres,Inc.
11. Ghazali, N. H. C. & Zakaria, E. (2011). Students' Procedural and Conceptual Understanding of Mathematics. *Australian Journal of Basic and Applied Sciences,* 5, 7, 684-691.
12. Hiebert, J. & Carpenter, T. P. (1992). *Learning and teaching with understanding.*

Handbook of Research in Teaching and Learning of Mathematics. Grouws D. W ed. New York: Macmilan.

13. House, J. D. & Telese, J. A. (2008). Relationships between students and instructional factors and algebra achievement of students in United States and Japan: An analysis of TIMSS 2003 data. *Educational Research and Evaluation,* 14, 1, 101-112.
14. Lerman, S. (2014). *Encyclopedia of Mathematics Education.* London: Springer.
15. Lopez, L. (1996). *Helping at-risk students solve mathematical word problems through the use of direct instruction and problem solving strategies,* Orlando: Unpublished Thesis.
16. Luneta, K. (2015). Understanding students' misconceptions: An analysis of final Grade 12 examination questions in geometry. *Pythagoras,* 36, 1, 1-11.
17. Marchionda, H. (2006). *Preservice teacher procedural and conceptual understanding of fractions and the effects of inquiry based learning on this understanding,* Clemson University: Unpublished Doctoral Dissertation.
18. McCormick, R. (1997). Conceptual and Procedural Knowledge. *International Journal of Technology and Design Education 7,* 141-159.
19. Mohyuddin, R. G., & Khalil, U. (2016). *Misconceptions of Students in Learning Mathematics at Primary Level.* Lahore: University of Management and Technology.
20. Mousley, J. (2004). *An Aspect of Mathematical Understanding: The Notion of "Connected Knowing".* Bergen, International Group for the Psychology of Mathematics Education, 377–384.
21. Mulungye, M., O'Connor, M. & Ndethiu. (2016). Sources of student errors and misconceptions in algebra and effectiveness of classroom practice remediation in Machakos County- Kenya. *Journal of Education and Practice,* 7, 10, 31-33.
22. Narayanan, L. M. (2007). *Analysis of Error in Addition and Subtraction of Fraction among Form 2,* Universiti Malaya: Unpublished Thesis.
23. Nathan, M. J. & Koellner, K. (2007). A Framework for understanding and cultivating the transition from arithmetic to algebraic reasoning. *Mathematical Thinking and Learning,* 9, 3, 179-192.
24. NCTM. (2000). *Principles and Standards for School Mathematics.* Reston, VA: The National Council of Teachers of Mathematics, Inc..
25. Ndemo, O., & Ndemo, Z. (2018). Secondary School Students' Errors and Misconceptions in Learning Algebra. *Journal of Education and Learning (EduLearn), 12*(4), 690-701. doi:10.11591/edulearn.v12i4.9956
26. Norasiah, A. (2002). *Error type diagnosis in learning simultaneous equation,* s.l.: Unpublished .
27. Orhun, N. (2015). *Students' Mistakes And Misconceptions On Teaching Of Trigonometry.* 1 ed. Eskisehir: Anadolu University.
28. Pournara, C., Hodgen, J., Sanders, Y. & Adler, J. (2016). Learners' errors in secondary algebra: Insights from tracking a cohort from Grade 9 to Grade 11 on a diagnostic algebra test. *Pythagoras,* 37, 1, 1-10.
29. Reid, N. & Yang, M.-J. (2002). The solving of problems in chemistry: the more open-ended problems. *Research in Science and Technological Education,* 2, 1, 83-96.
30. Sarwadi, H., & Shahrill, M. (2014). Understanding Students' Mathematical Errors and Misconceptions: The Case of Year 11 Repeating Students. Mathematics Education Trends and Research, 1-10. doi:10.5899/2014/metr-00051
31. Sudihartinih, E., & Purniati, T. (2020). Students' Mistakes and Misconceptions on the Subject of Conics. International Journal of Education, 12(2), 92-129. doi:10.17509/ije.v12i2.19130
32. Surif, J., Ibrahim, N. H. & Mokhtar, M. (2012). *Conceptual and Procedural Knowledge in Problem Solving.* s.l., Elsevier, Ltd, 416-425.
33. Taconis, R., Ferguson-Hessler & Broekkamp, H. (2000). Science teaching problem solving: an overview of experimental work. *Journal of Research in Science Teaching,* 38, 4, 442-468.
34. Tambychik, T. & Meerah, T. S. M. (2010). *Students' Difficulties in Mathematics Problem-Solving.* Malacca, Elsevier Ltd,

142-151.

35. Tobias, S. (1993). *Overcoming math anxiety revised and expanded.* New York: Norton.

36. Usman, M. H. & Hussaini, M. M. (2017). Analysis of students' error in learning of trigonometry among Senior Secondary School students in Zaria Metropolis, Nigeria. *IOSR Journal of Mathematics,* 13, 2, 1-4.

37. Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal,* 17, 3, 91-112.

38. Wells, D. (1994). Anxiety, insight and appreciation. *Mathematics Teaching,* Volume 147, 8-11.

39. Wilkins, J. (2000). Preparing for the 21 st century: The status of quantitative literacy in the United States. *School Science and Mathematics,* 100, 8, 405-418.

40. Wright, J. E. (2014). *An investigation of factors affecting student performance in algebraic word problem solutions,* North Carolina: Gardner-Webb University.

Table 1. *Indicators for Types of Mathematical Connection Errors*

No.	Type of Mathematical Connection Errors	Indicators
1	Errors in connecting to conceptual knowledge (Type A)	Applying the concept of calculation incorrectly Using concepts to prior knowledge inappropriately
2	Errors in connecting to procedural knowledge (Type B)	Using improper trigonometric rules or formulas Making a mistake/ unable to do algebraic manipulation Making sign errors in count operations

Table 2. *Percentage of Students' Failures on Each Item*

Item		Description of errors	Percentage of students' errors
1	Type A	Relating knowledge of algebraic arithmetical operations (eg. $\frac{1}{sin\,x} - \frac{1}{sin\,x}(1 - sin^2x)$ produces 1-sin^2x)	5.56
	Type B	Relating their knowledge that 1- sin^2x is equal with cos^2x	13.89
		Relating distributive properties to change an algebraic form	8.33
2	Type A	Relating their knowledge of the algebraic rule (e.g. conclude that $\sqrt{1} = \sqrt{sin^2x + cos^2x}$ produces $1 = sin\,x + cos\,x$)	44.44
		Relating their knowledge of the algebraic rule (a cos x + tan x = sec x into a = -cos x - tan x+sec)	38.89
		Relating their knowledge about the concept of linear equation system	5.56
	Type B	Relating their knowledge that $sec^2x - tan^2x$ is equal with 1	52.78
		Relating distributive properties to change an algebraic form	11.11
3	Type B	Relating their knowledge of multiplication with conjugate pair to change an algebraic form	11.11

APPENDIX

INTERVIEW GUIDELINES

The purpose of the interview

Obtain information about student mathematical connection errors in solving trigonometric identity problems and obtain information about the causes of these difficulties.

Interview Conditions

1. The questions adapted to the problem-solving conditions of the students (writing and explanation)
2. If students have difficulty to understand certain questions, the interviewer can provide a guide to simpler questions without reducing the gist of the problem.

Interview Implementation

1. Provide trigonometric identity test questions.
2. Ask students to read the test questions carefully. Make sure students understand each question and give students time to work on it.
3. To dig up data regarding errors in connecting to conceptual knowledge (Type A) is done with
    a. Asking main questions on the subject, for example, "What is the reason you get this final result?"
    b. Give follow-up questions so that you can explore the information processing done by the subject in detail when using concepts in initial knowledge, for example, "In this section, how do you/ what reasons do you get results like this?"
    c. Give follow-up questions so that you can explore the information processing carried out by the subject in detail when applying the concept of calculation, for example, "Please pay attention to the calculations in this section, is there a miscalculation?"
4. To dig up data regarding errors in connecting to procedural knowledge (Type B) done with
    a. Asking main questions on the subject, for example, "How will you solve the problem?"
    b. Give follow-up questions so that you can explore the information processing done by the subject in detail at the stage of determining trigonometric rules or formulas, for example, "What are the trigonometric identities that you use to solve the problem?"
    c. Give follow-up questions so that you can explore the mistakes made by the subject in detail, for example, " How do you change this algebraic form into another algebraic form?"
    d. Give follow-up questions so that you can explore the sign mistake made by the subject in detail, for example, “Is the sign of the operation/operation of your calculation correct?"