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MÉTODO CAS WAVELETS PARA RESOLVER O SISTEMA DE DIFUSÃO DE REAÇÃO E COMPARAR COM O MÉTODO (G.F.E)

CAS WAVELETS METHOD TO SOLVE REACTION-DIFFUSION SYSTEM AND COMPARE IT WITH (G.F.E) METHOD

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RESUMO

A análise de Onduleta ou ondaleta (em inglês*: wavelet*) desempenha um papel de destaque em vários campos das disciplinas científicas. Particularmente, as wavelets são usadas com muito sucesso na análise de sinais para representação e segmentação de formas de onda, análise de frequência-tempo e algoritmos rápidos nas equações de propagação e reação. Esta pesquisa teve como objetivo orientar os pesquisadores a usar Cos e Sin (CAS) para aproximar a solução do sistema de equações diferenciais parciais. Este método foi aplicado com sucesso para resolver um sistema acoplado de sistemas não-lineares de difusão de reação onde foi demonstrado que o método de wavelets CAS é bastante capaz e adequado para encontrar soluções exatas uma vez que a consistência do método oferece uma aplicabilidade mais ampla onde a ideia principal é transformar equações diferenciais parciais não lineares complexas em sistemas de equações algébricas, fáceis de manusear e encontrar uma solução numérica para elas. Ao comparar as soluções numéricas dos métodos de elementos finitos CAS e Galerkin, a solução de sistemas não-lineares de difusão de reação usando as wavelets do CAS para todos os valores de x é precisa, confiável, robusta, promissora e chega rapidamente à solução exata. Quando os parâmetros ε_1 e ε_2 estão crescendo e com L diminuindo, o método CAS converge rapidamente para soluções em estado estacionário (quanto menos L, mais precisa é a solução). Ele está convergindo para soluções de estado estacionário mais rapidamente e perde etapas ao longo do tempo. Além disso, os resultados também mostram que a solução das wavelets CAS é mais confiável e mais rápida em comparação com os elementos finitos de Galerkin (G.F.E).

Palavras-chave: *Sistema acoplado não-linear PDE, método de elementos finitos de Galerkin (G.F.E), solução numérica, wavelets Haar.*

ABSTRACT

Wavelet analysis plays a prominent role in various fields of scientific disciplines. Mainly, wavelets are very successfully used in signal analysis for waveform representation and segmentation, time-frequency analysis, and fast algorithms in the propagation equations and reaction. This research aimed to guide researchers to use Cos and Sin (CAS) to approximate the solution of the partial differential equation system. This method has been successfully applied to solve a coupled system of nonlinear Reaction-diffusion systems. It has been shown CAS wavelet method is quite capable and suited for finding exact solutions once the consistency of the method gives wider applicability where the main idea is to transform complex nonlinear partial differential equations into algebraic equation systems, which are easy to handle and find a numerical solution for them. By comparing the numerical solutions of the CAS and Galerkin finite elements methods, the answer of nonlinear Reaction-diffusion systems using the CAS wavelets for all $\hat{t}$ and x values is accurate, reliable, robust, promising, and quickly arrives at the exact solution. When parameters ε_1 and ε_2 are growing and with L decreasing, then the CAS method converges to steady-state solutions quickly (the less L, the more accurate the solution). It is converging towards steady-state solutions faster than and loses steps over time. Moreover, the results also show that the solution of the CAS wavelets is more reliable and faster compared to the Galerkin finite elements (G.F.E).

Keywords: *Coupled of nonlinear PDE system, Galerkin finite elements method (G.F.E), Numerical solution, Haar wavelets.*

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1. INTRODUCTION:

"Reaction-diffusion systems lead to the formation of tissue based on process simulations and nonlinear interactions, which were proposed as a model for the formation of a biological pattern. Alan Turing (1952) proposed the first chemical mechanism for calculating pattern formation in biological morphology, assuming tissue formation and chemical reaction and the amount of rate that depends on their concentration. These mechanisms are called reaction and diffusion systems" (Turing, 1952; Lepik, 2007; Qasim and Salim, 2019).

Wavelet analysis plays a prominent role in several areas of mathematics. The wavelets are utilized in the solution of all types of equations. These wavelets possess many remarkable properties. They studied intensively, and their connections with different branches of study, like convex and numerical analysis, were investigated. Furthermore, the Sin and Cos for the (CAS) wavelets used, to approximate the solution of linear integral equations (Yousefi and Banifatemi, 2006), Volterra integral equations of the second kind (Adibi and Assari,2009), and optimal control systems by time-dependent coefficients (Abualrub et al., 2009).

Convert differential equation into an algebraic one is the main characteristic of the operational method and the integration is a core of the functional matrix starting by the integral property of the basic orthonormal matrix, $\zeta(\hat{t})$ can write the following approximation:

$$\underbrace{\int_0^{\hat{t}}\int_0^{\hat{t}}\int_0^{\hat{t}}\cdots\cdots\int_0^{\hat{t}}}_{k}\psi(\hat{t})(d)^k \cong Q_\psi^k\psi(\hat{t}) \qquad \text{(Eq.1)}$$

Where $\psi(\hat{t})=[\bar{\psi}_0(\hat{t})\ \ \bar{\psi}_1(\hat{t})\ \ \ldots\ \ \bar{\psi}_{m-1}(\hat{t})]^T$ in which the elements $\bar{\psi}_0(\hat{t}),\ \bar{\psi}_1(\hat{t}),\ \ldots\ ,\bar{\psi}_{m-1}(\hat{t})$ are the discrete representation of essential functions which are orthogonal on the interval [0,1) and Q_ψ is the operational matrix for the integration of $Q(\hat{t})$ [10] (Wu and Chen, 2003).

Yousefi and Banifatemi, 2006, suggested introducing the CAS waves first. CAS wavelets approximation method utilized to reduce the Fredholm integral equations to the solution of algebraic equations. The CAS wavelet operational matrix P of integration will be first presented, and a general procedure to generate this matrix P is given. CAS wavelet approximating method utilized to reduce the integro-differential equation to the algebraic equations (Han and Shang, 2007). A wavelet-based method for evaluating the modal optimal control and trajectory of the bilinear system is proposed. The technique employs finite CAS wavelets to approximate modal control and state variables (Taher et al., 2009).

Presenting a computational method for solving nonlinear Fredholm integral equations: of the second kind which based on the use of Haar wavelets, for fractional-order which based on CAS (Cosine and Sine) wavelets, A new approach using CAS wavelets in combination with the collocation technique is proposed (Babolian and Shahsavaran, 2009; Saeedi et al., 2011; Ezzati and Najafalizadeh, 2012)

For fractional equations and wave waves, CAS: Using the series of CAS waves with an operational matrix to reduce fractional differential equations to the algebraic equation system. The solution to this system gives the approximate solution of the truncated finite "2K (2M + 1)" (Mingxu et al., 2013). A mathematical method is presented for the numerical solution of a class of differential equations with a weak single nucleus of a fractional system dependent on the Sin and Cos (CAS) waves and mass pulse functions (Mingxu and Jun, 2014). A novel numerical scheme for the approximate solutions of linear as well as nonlinear ordinary differential equations of fractional order with boundary conditions, this method combines Cosine and Sine (CAS) wavelets with Green function, called the Green-CAS method (Muhammad et al., 2019).

Presented a computational method for resolving integral boundary equations for singular logarithmic kernels, which occur as reformulations of a boundary value problem for Laplace's equation (Shamooshakya et al., 2014). The solution of time-varying systems obtained by using CAS wavelets, this method is based upon expanding various time functions in the system as their truncated CSA wavelets. The operational matrices of integration are utilized to reduce the resolution of time-varying systems to the solution of algebraic equations (Xiangyu, 2014). A based on CAS wavelets presented quadrature rules for the numerical solution of double and triple integrals with variable limits of integration (Rezabeyk and Maleknejad, 2015). Present a computational method for solving boundary integral equations with singular logarithmic kernels that occur as reformulations of a boundary value problem for Laplace's equation (Fathizadeh et al., 2017). Proposed a new operational matrix of differentiation using CAS wavelets, with the aid of

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these matrices, the CAS wavelets method (CASWM) is developed for the integrodifferential equations (Shiralashetti and Kumbinarasaiah, 2019). Solved this system numerically by finite elements method (G.F.E) method, and found that (G.F.E) method is faster than (F.D) method to reach equilibrium state for all $\hat{t}$ and x values (Manaa and Qasim, 2019).

Therefore, this study aimed to highlight the CAS wavelets method and test the strength of this method by comparing the solution with a widely known and well-method, which is the Galerkin finite element method.

2. MATERIALS AND METHODS:

2.1. CAS wavelet method

CAS wavelets method can be written as: $\tau_{\gamma,m}(\chi)=\tau(\rho,\gamma,m,\chi)$ Where $\gamma=1,2.....2^{\rho}$, ρ ,m a positive integer and χ is the normalized time.

Orthonormal CAS wavelets are defined on the [0,1] by:

$$\tau_{\gamma,m}(\chi)=\begin{cases} 2^{\frac{\rho}{2}} CAS_m(2^{\rho}\chi-\gamma+1) & for \quad \frac{\gamma-1}{2^{\rho}}\le\chi<\frac{\gamma}{2^{\rho}} \\ 0 & otherwise \end{cases}$$

(Eq. 2)

Where $CAS_m(\chi)=cos(2\pi m\chi)+\sin(2\pi m\chi)$, and $m\in\{-M,.....,M\}$ the CAS wavelets are orthonormal for weight function $W(x)=1$.

The CAS wavelets have the following form with m=0:

$$\tau_{\gamma,0}(\chi)=2^{\frac{\rho}{2}}G_{\gamma}(\chi)=2^{\frac{\rho}{2}}\begin{cases} 1 & \frac{\gamma-1}{2^{\rho}}\le\chi<\frac{\gamma}{2^{\rho}} \\ 0 & otherwise \end{cases}$$

(Eq. 3)

where $\{G_{\gamma}(\chi)\}_{\gamma=1}^{2^{\rho}}$ is Block –Pulse functions (basis set) on the close interval [0,1] (Lepik, 2007).

It can be defined as the integral of (Eq. 2) by CAS wavelets analytically as:

$$P_{2^{\rho}(2M+1),1}(\chi)=\int_0^{\chi}\tau_{\gamma,m}(\chi')\,d\chi'$$

$$=\begin{cases} 0 & 0\le\chi<\frac{\gamma-1}{2^{\rho}} \\ \int_{\frac{\gamma-1}{2^{\rho}}}^{x} 2^{\frac{\rho}{2}}(\varpi)d\chi' & \frac{\gamma-1}{2^{\rho}}\le\chi<\frac{\gamma}{2^{\rho}} \\ \int_{\frac{\gamma-1}{2^{\rho}}}^{\frac{\gamma}{2^{\rho}}} 2^{\frac{\rho}{2}}(\varpi)d\chi' & \frac{\gamma}{2^{\rho}}\le\chi<1 \end{cases}$$

Where

$$\varpi=\sin(2\pi m(2^{\rho}\chi'-\gamma+1)+cos(2\pi m(2^{\rho}\chi'-\gamma+1)))$$

Then

$$P_{2^{\rho}(2M+1),1}(\chi)=\begin{cases} 0 & 0\le\chi<\frac{\gamma-1}{2^{\rho}} \\ 2^{\frac{\rho}{2}}\frac{1}{\varphi}(\sin(2\pi\sin(2^{\rho}\chi-\gamma+1))- cos(2\pi m(2^{\rho}\chi-\gamma+1)))-2^{\frac{\rho}{2}}\frac{-1}{\varphi}, & \frac{\gamma-1}{2^{\rho}}\le\chi<\frac{\gamma}{2^{\rho}} \\ 2^{\frac{\rho}{2}}\frac{1}{\varphi}(\sin(2\pi m))-cos(2\pi m))-2^{\frac{\rho}{2}}\frac{-1}{\varphi}, & \frac{\gamma}{2^{\rho}}\le\chi<1 \end{cases}$$

Where $\varphi=2^{\rho+1}\pi m$

Founding the integration n times as:

$$P_{\lambda,u}(x)=\begin{cases} 0 & 0\le\chi<\frac{\gamma-1}{2^{\rho}} \\ 2^{\frac{\rho}{2}}\frac{(-1)^{f_u}}{(\varphi)^u}\cos(2\pi m(2^{\rho}\chi-\gamma+1)) +2^{\frac{\rho}{2}}\frac{(-1)^{e_u}}{(\varphi)^u}\sin(2\pi m(2^{\rho}\chi-\gamma+1)) -\sum_{jj=0}^{u-1}2^{\frac{\rho}{2}}\frac{1}{jj!}\frac{(-1)^{f_u}}{(\varphi)^{u-jj}}\left(\chi-\frac{\gamma-1}{2^{\rho}}\right)^{jj} & \frac{\gamma-1}{2^{\rho}}\le\chi<\frac{\gamma}{2^{\rho}} \\ \sum_{j=0}^{u-1}\frac{1}{j!}\left(\chi-\frac{\gamma}{2^{\rho}}\right)^{j} & \frac{\gamma}{2^{\rho}}\le\chi<1 \\ \left(2^{\frac{\rho}{2}}\frac{(-1)^{f_u}}{(\varphi)^u}cos(2\pi m)+2^{\frac{\rho}{2}}\frac{(-1)^{e_u}}{(\varphi)^u}\sin(2\pi m) -\sum_{jj=0}^{u-1}2^{\frac{\rho}{2}}\frac{1}{jj!}\frac{(-1)^{f_u}}{(\varphi)^{u-jj}}\left(\frac{1}{2^{\rho}}\right)^{jj}\right) & \end{cases}$$

(Eq. 4)

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Where $\varphi = 2^{\rho+1}\pi m$, and

$$f_u = \begin{cases} 0 & if\ \ u = 3,4,7,8,11,12 \ldots\ldots \\ 1 & if\ \ u = 1,2,5,6,9,10, \ldots\ldots \end{cases}$$

And

$$e_u = \begin{cases} 0 & if\ \ u = 1,4,5,8,9,12, \ldots\ldots \\ 1 & if\ \ u = 2,3,6,7,10,11, \ldots\ldots \end{cases}$$

If u=0 corresponds to CAS function $\tau_{\gamma,m}(\chi)$ in (Eq. 2). To solve boundary value problems, substitute $\chi = G$ in (Eq. 4). In some instances where u=1, finding (Lepik, 2007):

$$P_{\lambda,u}(x) = \begin{cases} 0 & 0 \le \chi < \frac{\gamma-1}{2^\rho} \\ \begin{cases} +2^{\frac{\rho}{2}} \frac{1}{\varphi} \sin(2\pi m(2^\rho G - \gamma + 1)) - \cos(2\pi m(2^\rho G - \gamma + 1)) \\ -2^{\frac{\rho}{2}} \frac{-1}{\varphi} \end{cases} & \frac{\gamma-1}{2^\rho} \le \chi < \frac{\gamma}{2^\rho} \\ \begin{cases} 2^{\frac{\rho}{2}} \frac{1}{(\varphi)} (\sin(2\pi m) - cos(2\pi m)) - 2^{\frac{\rho}{2}} \frac{-1}{\varphi} \end{cases} & \end{cases}$$

2.2. Approximation CAS Function

Expanded $\upsilon(\chi)$ by CAS wavelets such that:

$$\upsilon(\chi) = \sum_{\lambda=0}^{\infty} z_\lambda \, \tau_{\lambda,m}(\chi) \quad , \lambda = 0,1,2,3.....,N \qquad \text{(Eq. 5)}$$

Hence:

$$Z_0 = \int_0^1 \upsilon(\chi)\tau_0(\chi)d\chi,$$

$$Z_\lambda = 2^j \int_0^1 \upsilon(\chi)\tau_\lambda(\chi)d\chi \qquad \text{(Eq. 6)}$$

$$\lambda = 2^j + \rho, \quad j \ge 0, \quad 0 \le \rho < 2^j, \quad \chi \in [0,1)$$

The integral square error E minimized as:

$$E = \int_0^1 \left[\upsilon(\chi) - \sum_{\lambda=0}^{2^\rho - 1} z_\lambda \tau_\lambda(\chi) \right]^2 d\chi, \qquad \text{(Eq. 7)}$$

$$\rho = 0,1,2.....N$$

Any function $\upsilon(\chi) \in L^2[0,1)$ may be expanded using CAS wavelets as:

$$\upsilon(\chi) = \sum_{\lambda=0}^{2^\rho} z_{\lambda,m} \, \tau_{\lambda,m}(\chi) = z^T \, \tau(\chi) \qquad \text{(Eq. 8)}$$

Where $m \in [-M, M]$ and $Z, \tau(x)$ are matrices given by:

$$Z = [Z_{1,(-M)}, Z_{1,(-M+1)}, \ldots\ldots, Z_{1,M}, \\ Z_{2^\rho,(-M)}, Z_{2^\rho,(-M+1)}, \ldots\ldots, Z_{2^\rho,M}]^T$$

$$\tau(x) = [\tau_{1,(-M)}(\chi), \tau_{1,(-M+1)}(\chi), \tau_{1,(M)}(\chi), \ldots\ldots, \\ ,\ldots, \tau_{2,(=M)}(\chi), \tau_{2^\rho,(-M)}(\chi), \ldots\ldots, \tau_{2^\rho,(M)}(\chi)]^T$$

; T transposes (Hariharan and Kannan, 2010).

2.3. Mathematical Model

Applying CAS wavelets method to solve nonlinear reaction-diffusion system which has the form (Kan-on, 1997):

$$\frac{\partial \upsilon}{\partial t_*} = \varepsilon_1 \frac{\partial^2 \upsilon}{\partial \chi_*^2} + \upsilon\left[\alpha_1(\chi_*) - \beta_1 \upsilon - \delta_1 \sigma\right]$$

$$\frac{\partial \sigma}{\partial t_*} = \varepsilon_2 \frac{\partial^2 \sigma}{\partial \chi_*^2} - \sigma\left[\alpha_2(\chi_*) - \delta_2 \upsilon\right] \qquad \text{(Eq. 9)}$$

With the initial and boundary conditions:

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$$\upsilon(\chi_*,0)=\upsilon_0(\chi)$$
$$\sigma(\chi_*,0)=\sigma_0(\chi)$$
$$\frac{\partial\upsilon}{\partial\chi_*}=\frac{\partial\sigma}{\partial\chi_*}=\frac{\partial\upsilon}{\partial\chi_*}(a,t)=\frac{\partial\upsilon}{\partial\chi}(b,t)=0$$
$$\chi_*\in[a,b]$$
$$t\geq 0$$

(Eq. 10)

$\upsilon(\chi,\hat{t})$ represents the reaction type, $\sigma(\chi,\hat{t})$ represents the diffusion type. $\varepsilon_1,\varepsilon_2,\beta_1,\delta_1,\delta_2$ are positive constants. $\alpha_1(\chi),\alpha_2(\chi)$ are the rates of growth of aspects, υ_0 and σ_0 are constant interior solutions of (Eq. 9).

First normalize with regard to $\chi_*\in[a,b]$ because CAS wavelets method defined for $\chi\in[0,1]$, the variables changing as (Lou et al., 2006):

$$\chi=\frac{1}{L}(\chi_*-a),\quad \hat{t}=\hat{t}_*-0,\qquad L=b-a$$

(Eq. 9), (Eq. 10) becomes:

$$\frac{\partial\upsilon}{\partial\hat{t}}=\frac{\varepsilon_1}{L^2}\frac{\partial^2\upsilon}{\partial\chi^2}+\upsilon\left[\alpha_1(L\chi+a)-\beta_1\upsilon-\delta_1\sigma\right]$$
$$\frac{\partial\sigma}{\partial\hat{t}}=\frac{\varepsilon_2}{L^2}\frac{\partial^2\sigma}{\partial\chi^2}-\sigma\left[\alpha_2(L\chi+a)-\delta_2\upsilon\right]$$

(Eq. 11)

And

$$\upsilon(\chi,0)=\upsilon_0(\chi)\quad,\quad \sigma(\chi,0)=\sigma_0(\chi)$$
$$\frac{\partial\upsilon}{\partial\chi}=\frac{\partial\sigma}{\partial\chi}=\frac{\partial\upsilon}{\partial\chi}(0,\hat{t})=\frac{\partial\upsilon}{\partial\chi}(1,\hat{t})=0$$
$$x\in[0,1]$$
$$\hat{t}\geq 0$$

(Eq. 12)

Suppose that, $\upsilon_{\chi\chi}(\chi,\hat{t}),\sigma_{\chi\chi}(\chi,\hat{t})$ can extend the range of CAS waves as follows:

$$\upsilon_{\chi\chi}(\chi,\hat{t})=\sum_{\lambda=0}^{2^\rho}\sum_{m=-M}^{M}z_s(\lambda)\tau_\lambda(x)=z_m^T\tau_m(x)\qquad t\in(\hat{t}_s,\hat{t}_{s+1}]$$

$$\sigma_{\chi\chi}(\chi,\hat{t})=\sum_{\lambda=0}^{2^\rho}\sum_{m=-M}^{M}z_s(\lambda)\tau_\lambda(x)=z_m^T\tau_m(x)\qquad t\in(\hat{t}_s,\hat{t}_{s+1}]$$

(Eq. 13)

Where the elements $z_{(m)}^T$ are constant in the subinterval

$$t_s=(s-1)\Delta\hat{t}\ ,\ \Delta\hat{t}=\frac{T}{N}\ \hat{t}\in(\hat{t}_s,\hat{t}_{s+1}]$$
$$s=1,2,3,\ldots\ldots N,\hat{t}\in(0,T)$$

Integrating (Eq. 13) finding:

$$\upsilon_{\chi\chi}(\chi,\hat{t})=(\hat{t}-\hat{t}_s)z_m^T\tau_m(\chi)+\upsilon_{\chi\chi}(\chi,\hat{t}_s)$$
$$\sigma_{\chi\chi}(\chi,\hat{t})=(\hat{t}-\hat{t}_s)z_m^T\tau_m(\chi)+\sigma_{\chi\chi}(\chi,\hat{t}_s)$$

(Eq. 14)

$$\upsilon_\chi(\chi,\hat{t})=(\hat{t}-\hat{t}_s)z_m^T P_{\lambda,1}(\chi)+[\upsilon_\chi(\chi,\hat{t}_s)-\upsilon_\chi(0,\hat{t}_s)]+\upsilon_\chi(0,\hat{t})$$
$$\sigma_\chi(\chi,\hat{t})=(\hat{t}-\hat{t}_s)z_m^T P_{\lambda,1}(\chi)+[\sigma_\chi(\chi,\hat{t}_s)-\sigma_\chi(0,\hat{t}_s)]+\sigma_\chi(0,\hat{t})$$

(Eq. 15)

$$\upsilon(\chi,\hat{t})=(\hat{t}-\hat{t}_s)z_m^T P_{\lambda,2}(\chi)+[\upsilon(\chi,\hat{t}_s)-\upsilon(0,\hat{t}_s)]+\chi[\upsilon(0,\hat{t})-\upsilon(0,\hat{t}_s)]+\upsilon(0,\hat{t})$$
$$\sigma(\chi,\hat{t})=(\hat{t}-\hat{t}_s)z_m^T P_{\lambda,2}(\chi)+[\sigma(\chi,\hat{t}_s)-\sigma(0,\hat{t}_s)]+\chi[\sigma(0,\hat{t})-\sigma(0,\hat{t}_s)]+\sigma(0,\hat{t})$$

(Eq. 16)

The differential of equation (Eq.16) with respect $\hat{t}$ can be writing:

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$$\upsilon^{\bullet}(\chi,\hat{t}) = z_m^T P_{\lambda,2}(\chi) + x\upsilon_x^{\bullet}(0,\hat{t}) + \upsilon^{\bullet}(0,\hat{t})$$
$$\sigma^{\bullet}(\chi,\hat{t}) = z_m^T P_{\lambda,2}(\chi) + \chi\sigma_{\chi}^{\bullet}(0,\hat{t}) + \sigma^{\bullet}(0,\hat{t})$$

(Eq. 17)

It can be reducing the boundary condition $\upsilon_{\chi}^{\bullet}(0,\hat{t})$ and $\upsilon^{\bullet}(0,\hat{t})$ in equation (Eq. 17) such that:

$$\upsilon^{\bullet}(0,\hat{t}) = \frac{\upsilon(0,\hat{t}) - \upsilon(0,\hat{t}_s)}{(\hat{t} - \hat{t}_s)}$$

Then (Eq. 17) for $\upsilon^{\bullet}(\chi,\hat{t})$ becomes:

$$\upsilon^{\bullet}(\chi,\hat{t}) = z_m^T P_{\lambda,2}(\chi) + \chi\left[\frac{\upsilon_{\chi}(0,\hat{t}) - \upsilon_{\chi}(0,\hat{t}_s)}{(\hat{t} - \hat{t}_s)}\right] + \left[\frac{\upsilon(0,t) - \upsilon(0,\hat{t}_s)}{(\hat{t} - t_s)}\right]$$

(Eq. 18)

Now, by substitute equations (Eq. 14) - (Eq. 18) in (Eq. 11), then get:

$$z_m^T P_{\lambda,2}(\chi) + \frac{\chi}{\Delta\hat{t}}\left[\upsilon_{\chi}(0,\hat{t}) - \upsilon_{\chi}(0,\hat{t}_s)\right] - \frac{1}{\Delta t}\left[\upsilon(0,\hat{t}) - \upsilon(0,\hat{t}_s)\right] = \frac{\varepsilon_1\Delta\hat{t}}{L^2} z_m^T \tau\upsilon(\chi,\hat{t}) + \frac{\varepsilon_1}{L^2}\upsilon_{\chi\chi}(\chi,\hat{t}) + \upsilon(\chi,\hat{t}_s)\left[\alpha_1(\chi L + a) - \beta_1\upsilon(\chi,\hat{t}_s) - \delta_1\sigma(\chi,\hat{t}_s)\right]$$

$$z_m^T P_{\lambda,2}(\chi) + \frac{x}{\Delta\hat{t}}\left[\sigma_{\chi}(0,\hat{t}) - \sigma_{\chi}(0,\hat{t}_s)\right] - \frac{1}{\Delta t}\left[\sigma(0,\hat{t}) - \sigma(0,\hat{t}_s)\right] = \frac{\varepsilon_2\Delta\hat{t}}{L^2} z_m^T \tau\sigma(\chi,\hat{t}) + \frac{\varepsilon_2}{L^2}\sigma_{\chi\chi}(\chi,\hat{t}) - \sigma(\chi,\hat{t}_s)\left[\alpha_2(\chi L + a) - \delta_2\upsilon(\chi,\hat{t}_s)\right]$$

(Eq. 19)

The (Eq.19) is a linear system. The problem was found to be resolved (Eq. 16).

3. RESULTS AND DISCUSSION:

3.1. Reducing the order boundary conditions:

Integrating equation (Eq. 17) to calculate unknown term such as:

$$\int_0^1 \upsilon_{\chi}(\chi,t) = \int_0^1 (\hat{t} - \hat{t}_s) z_m^T P_{\lambda,1}(\chi)\, d\chi + \int_0^1 \upsilon_{\chi}(\chi,\hat{t}_s)\, d\chi + \int_0^1 [\upsilon_{\chi}(0,\hat{t}) - \upsilon_{\chi\chi}(0,\hat{t}_s)]\, d\chi$$

$$\Rightarrow \upsilon(1,\hat{t}) - \upsilon(0,\hat{t}) = (\hat{t} - \hat{t}_s) z_m^T D_{\lambda,1}(\chi) + [\upsilon(1,\hat{t}_s) - \upsilon(0,\hat{t}_s)] + [\upsilon_{\chi}(0,\hat{t}) - \upsilon_{\chi}(0,\hat{t}_s)]$$

$$\Rightarrow [\upsilon_{\chi}(0,\hat{t}) - \upsilon_{\chi}(0,\hat{t}_s)] = -(\hat{t} - \hat{t}_s) z_m^T D_{\lambda,1}(\chi) + [\upsilon(1,\hat{t}) - \upsilon(1,\hat{t}_s)] - [\upsilon(0,\hat{t}) - \upsilon(0,\hat{t}_s)]$$

(Eq. 20)

Where $D_{\lambda,1}(\chi) = \int_0^1 P_{\lambda,1}(\chi) d\chi$

By substitute (Eq. 20) in equations (Eq. 14) - (Eq. 18) about $\upsilon(\chi,\hat{t})$ we get:

$$\upsilon_{\chi\chi}(\chi,\hat{t}) = (\hat{t} - \hat{t}_s) z_m^T \tau_m(\chi) + \upsilon_{\chi\chi}(\chi,\hat{t}_s)$$

(Eq. 21)

$$\upsilon_{\chi}(\chi,\hat{t}) = (\hat{t} - \hat{t}_s) z_m^T P_{\lambda,1}(\chi) - (\hat{t} - \hat{t}_s) z_m^T D_{i,1}(\chi) + [\upsilon(1,\hat{t}) - \upsilon(1,\hat{t}_s)] - [\upsilon(0,\hat{t}) - \upsilon(0,\hat{t}_s)] + \upsilon_{\chi}(0,\hat{t}_s)$$

(Eq. 22)

$$\upsilon(\chi,\hat{t}) = (\hat{t} - \hat{t}_s) z_m^T P_{\lambda,2}(x) + [\upsilon_{\chi}(\chi,\hat{t}_s) - \upsilon_{\chi}(0,\hat{t}_s)] + \chi[-(\hat{t} - \hat{t}_s) z_m^T D_{i,1}(x) + [\upsilon(1,\hat{t}) - \upsilon(1,\hat{t}_s)] - [\upsilon(0,\hat{t}) - \upsilon(0,\hat{t}_s)] + \upsilon(0,\hat{t}_s)$$

(Eq. 23)

Now, differential (Eq. 23) by respect $\hat{t}$, then get:

$$\upsilon^{\bullet}(\chi,\hat{t}) = z_m^T P_{\lambda,2}(\chi) + \frac{1}{(\hat{t} - \hat{t}_s)}[\upsilon_x(\chi,\hat{t}_s) - \upsilon_x(0,\hat{t}_s)] + \frac{\chi}{(\hat{t} - \hat{t}_s)}[-(\hat{t} - \hat{t}_s) z_m^T D_{\lambda,1}(\chi) + \frac{1}{(\hat{t} - \hat{t}_s)}[\upsilon(1,\hat{t}) - \upsilon(1,\hat{t}_s)] - [\upsilon(0,\hat{t}) - \upsilon(0,\hat{t}_s)]$$

(Eq. 24)

Now, by substitute equations (Eq. 21) - (Eq. 24) in (Eq. 11), then get:

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$$z_m^T P_{\lambda,2}(\chi) + \frac{1}{\Delta\hat{t}}[\upsilon_\chi(\chi,\hat{t}_s) - \upsilon_\chi(0,\hat{t}_s)] + \frac{\chi}{\Delta\hat{t}}[-\Delta t\, z_m^T D_{i,1}(\chi)]$$
$$+ \frac{1}{\Delta\hat{t}}[\upsilon(1,\hat{t}) - \upsilon(1,\hat{t}_s)] - [\upsilon(0,\hat{t}) - \upsilon(0,\hat{t}_s)] = \frac{\varepsilon_1 \Delta\hat{t}}{L^2}\, z_m^T \tau_m(\chi)$$
$$+ \frac{\varepsilon_1 \Delta\hat{t}}{L^2}\upsilon_{\chi\chi}(\chi,\hat{t}_s) + \upsilon(\chi,\hat{t}_s)[\alpha_1(\chi) - \beta_1\upsilon(\chi,\hat{t}_s) - \delta_1\sigma(\chi,\hat{t}_s)]$$

Similarly, find $\sigma(\chi,\hat{t}_s)$ then:

$$z_m^T P_{\lambda,2}(\chi) - xz_m^T D_{\lambda,1}(\chi) - \frac{\varepsilon_1 \Delta\hat{t}}{L^2} z_m^T \tau_m(\chi) = -\frac{1}{\Delta\hat{t}}[\upsilon_\chi(\chi,\hat{t}_s)$$
$$- \upsilon_\chi(0,\hat{t}_s)] - \frac{1}{\Delta t}[\upsilon(1,\hat{t}) - \upsilon(1,\hat{t}_s)] + [\upsilon(0,\hat{t})$$
$$- \upsilon(0,\hat{t}_s)] + \frac{\varepsilon_1 \Delta\hat{t}}{L^2}\upsilon_{\chi\chi}(\chi,\hat{t}_s) + \upsilon(\chi,\hat{t}_s)$$
$$[\alpha_1(\chi) - \beta_1\upsilon(\chi,\hat{t}_s) - \delta_1\sigma(\chi,\hat{t}_s)]$$

$$z_m^T P_{\lambda,2}(\chi) - xz_m^T D_{\lambda,1}(\chi) - \frac{\varepsilon_2 \Delta\hat{t}}{L^2} z_m^T \tau_m(\chi) = -\frac{1}{\varepsilon_1 \Delta\hat{t}}[\sigma_\chi(\chi,\hat{t}_s)$$
$$-\sigma_\chi(0,\hat{t}_s)] - \frac{1}{\Delta t}[\sigma(1,\hat{t}) - \sigma(1,\hat{t}_s)] + [\sigma(0,\hat{t}) - \sigma(0,\hat{t}_s)]$$
$$+ \frac{\varepsilon_2 \Delta\hat{t}}{L^2}\sigma_{\chi\chi}(\chi,\hat{t}_s) - \sigma(\chi,\hat{t}_s)[\alpha_2(\chi) - \delta_2\upsilon(\chi,\hat{t}_s)]$$

(Eq. 25)

3.2. Numerical results and Discussion

solve (Eq. 11) by using (Eq. 25) and the exact solution such that (Lou and et al., 2006):

$$\left.\begin{aligned} \upsilon(\chi,0) &= \frac{\alpha_1}{\beta_1} + \frac{1}{(\alpha_1-\alpha_2)\beta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi) \\ \sigma(\chi,0) &= \frac{1}{(\alpha_2-\alpha_1)\delta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi) \end{aligned}\right\}$$

(Eq. 26)

Where $\mu = \dfrac{\alpha_1 - \alpha_2}{\varepsilon_1 - \varepsilon_2} < 0, \qquad \alpha_1, \alpha_2 > 0$

$$\upsilon(\chi,0) = \upsilon_0(\chi)$$
$$\sigma(\chi,0) = \sigma_0(\chi)$$
$$\frac{\partial\upsilon}{\partial\chi} = \frac{\partial\sigma}{\partial\chi} = \frac{\partial\upsilon}{\partial\chi}(0,\hat{t}) = \frac{\partial\upsilon}{\partial\chi}(1,\hat{t}) = 0$$
$$0 \le \chi \le 1$$
$$t \ge 0$$

(Eq. 27)

Thus will found:

$$\left.\begin{aligned} \upsilon(\chi,\hat{t}_s) = \upsilon(\chi,0) &= \frac{\alpha_1}{\beta_1} + \frac{1}{(\alpha_1-\alpha_2)\beta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi) \\ \sigma(\chi,\hat{t}_s) = \sigma(\chi,0) &= \frac{1}{(\alpha_2-\alpha_1)\delta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi) \end{aligned}\right\}$$

$$\left.\begin{aligned} \upsilon_\chi(\chi,\hat{t}_s) = \upsilon(\chi,0) &= \frac{\partial}{\partial x}\frac{\alpha_1}{\beta_1} + \frac{1}{(\alpha_1-\alpha_2)\beta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi) \\ \sigma_\chi(\chi,\hat{t}_s) = \sigma(\chi,0) &= \frac{\partial}{\partial x}\left[\frac{1}{(\alpha_2-\alpha_1)\delta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi)\right] \end{aligned}\right\}$$

$$\left.\begin{aligned} \upsilon_{\chi\chi}(\chi,\hat{t}_s) = \upsilon(\chi,0) &= \frac{\partial^2}{\partial\chi^2}\left[\frac{\alpha_1}{\beta_1} + \frac{1}{(\alpha_1-\alpha_2)\beta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi)\right] \\ \sigma_{\chi\chi}(x,\hat{t}_s) = \sigma(\chi,0) &= \frac{\partial^2}{\partial\chi^2}\left[\frac{1}{(\alpha_2-\alpha_1)\delta_1}\delta_2 \sin(\sqrt{-\mu\varepsilon_1}\chi)\right] \end{aligned}\right\}$$

(Eq. 28)

Comparison of the numerical solution and the exact solution by us (Lou and et al., 2006):

$$\upsilon(\chi,0) = \frac{1}{10}\sin^2\left(\frac{24}{10}\pi\chi\right) + \sin^2\left(\frac{-5}{100}\pi\chi\right) \qquad \chi \in [0,L]$$
$$\sigma(\chi,0) = \frac{1}{10}\sin^2\left(\frac{24}{10}\pi\chi\right) + \sin^2\left(\frac{-5}{100}\pi\chi\right) \qquad \chi \in [0,L]$$

Where L=0.5 and time step $\Delta\hat{t} = 0.1, \varepsilon_1 = \varepsilon_2 = 0.001, \theta = 0.5, \alpha_1 = 1, \alpha_2 = 0.06,$ $\beta_1 = \delta_1 = \delta_2 = 1$ the results are given in Table 1 and Figures 1 and 2.

Where L=0.5 is step size and time step $\Delta\hat{t} = 0.1, \varepsilon_1 = \varepsilon_2 = 0.001, \theta = 0.5, \alpha_1 = 1, \alpha_2 = 0.01$, $\beta_1 = \delta_1 = \delta_2 = 0.0001$ and M=21 the results given in Table 2 and Figures 3 and 4.

Table 3 represents a comparison of the system's absolute errors (Eq. 11) between the CAS and finite element results with the exact solution. Noted the calculation of the CAS method in Table 2 We chose a value $\beta_1, \delta_1, \delta_2$ close to zero which is, $\beta_1 = \delta_1 = \delta_2 = 0.0001$, while we took the value $\beta_1 = \delta_1 = \delta_2 = 0$ in the calculation of the finite element method, because when calculating in the CAS method and when one of them $\beta_1 = \delta_1 = \delta_2 = 0$ an appears in the solution amount is not defined.

From solving nonlinear partial differential equations for Reaction-diffusion Systems numerically by using the CAS method. “This

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method found that it is better and faster for all the values for $\hat{t}$ and x. It also found that it is converging towards steady-state solutions faster than and loses steps over time". As well, the CAS method converges to an exact solution faster when growing $\varepsilon_1 and \varepsilon_2$ and decreasing L. When $\hat{t}$ is reduced, the solution will be quick to arrive at the real solution.

Finally, the comparison between the CAS methods and the finite elements was found the solution converged but clear preference for the CAS method on the finite element method, as shown in Tables 1 - 3 and Figures 1 - 4.

4. CONCLUSIONS:

CAS wavelet method has been successfully applied to solve a coupled system of nonlinear Reaction-diffusion systems. It has been shown that the CAS wavelets method is quite capable and suited for finding exact solutions. The consistency of the method gives this method wider applicability. By comparing the numerical solution of the nonlinear Reaction-diffusion system and using the CAS wavelets and finite element methods with the exact solution, it was found that the CAS wavelets method is the most accurate and fastest way to reach the exact solution. Maple 18 was used to carry out the computations. Finally, it is worth noting that this method is clear and concise, and can be applied to nonlinear PDF equations in engineering and applied science.

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Table 1. *A CAS wavelets, Galerkin finite element method and the exact solution or* $\upsilon(\chi,\hat{t}),\sigma(\chi,\hat{t})$ *when* $\hat{t}$ *=14.5,* $\hat{t}$ *=8 and comparison between them.*

NO.	$\lvert CAS - exact \rvert$ $\hat{t}$=14.5 $\upsilon(\chi,\hat{t})$	$\lvert G.F.E - exact \rvert$ $\hat{t}$=14.5 $\upsilon(\chi,\hat{t})$	$\lvert CAS - exact \rvert$ $\hat{t}$=8 $\sigma(\chi,\hat{t})$	$\lvert G.F.E - exact \rvert$ $\hat{t}$=8 $\sigma(\chi,\hat{t})$	$\lvert CAS - exact \rvert$ $\hat{t}$=9.5 $\upsilon(\chi,\hat{t})$	$\lvert G.F.E - exact \rvert$ $\hat{t}$=9.5 $\upsilon(\chi,\hat{t})$	$\lvert CAS - exact \rvert$ $\hat{t}$=13.5 $\sigma(\chi,\hat{t})$	$\lvert G.F.E - exact \rvert$ $\hat{t}$=13.5 $\sigma(\chi,\hat{t})$
1	8.19E-04	1.01E-03	0.00E+00	1.11E-02	7.60E-05	1.00E-04	1.09E-05	8.00E-04
2	7.43E-04	8.41E-04	9.04E-03	1.11E-02	7.89E-05	9.99E-05	2.54E-04	1.10E-03
3	3.76E-04	4.11E-04	5.10E-04	6.70E-02	1.05E-04	1.29E-04	2.13E-04	3.01E-04
4	7.79E-05	1.11E-04	6.19E-03	6.54E-03	8.60E-05	8.91E-05	1.65E-04	3.00E-04
5	6.63E-05	1.11E-04	2.63E-03	6.89E-03	8.62E-05	9.68E-05	8.99E-05	3.00E-04
6	2.05E-04	2.41E-04	2.52E-03	2.89E-03	1.06E-04	7.22E-05	2.29E-04	4.00E-04
7	5.74E-04	6.11E-04	2.49E-03	2.89E-03	1.05E-04	1.00E-04	1.60E-04	2.00E-04
8	5.88E-04	4.11E-04	2.33E-02	3.02E-02	1.05E-04	1.34E-04	2.13E-04	2.00E-04
9	7.97E-04	6.11E-04	5.20E-03	9.45E-03	1.38E-04	1.53E-04	3.94E-04	4.00E-04
10	7.98E-04	6.11E-04	1.13E-03	1.11E-03	8.35E-05	1.16E-04	4.12E-04	3.00E-04
11	4.14E-05	1.62E-04	7.73E-04	2.66E-03	5.21E-06	6.00E-05	2.72E-04	3.00E-04
12	3.03E-04	1.12E-04	2.31E-04	2.88E-04	1.51E-05	3.32E-05	5.22E-04	6.00E-04
13	1.77E-04	1.11E-04	4.68E-05	4.48E-04	5.39E-06	4.17E-05	3.20E-04	5.00E-04
14	1.84E-04	1.11E-04	3.32E-05	2.10E-04	5.24E-06	3.36E-05	2.19E-04	4.00E-04
15	7.42E-06	1.69E-04	3.17E-04	4.47E-04	5.18E-06	4.00E-07	2.13E-04	3.00E-04
16	9.14E-05	1.11E-04	1.61E-02	2.37E-02	3.94E-06	1.77E-05	2.42E-04	3.00E-04
17	4.65E-04	1.11E-04	3.05E-03	7.60E-03	3.39E-06	1.42E-05	3.28E-04	4.00E-04
18	2.03E-05	1.11E-04	5.35E-04	9.89E-04	3.26E-06	3.43E-05	9.24E-04	1.10E-03
19	2.30E-05	2.11E-05	2.01E-03	2.03E-03	3.50E-06	3.98E-05	8.67E-04	1.02E-03
20	3.40E-05	9.92E-05	2.00E-04	6.19E-04	1.39E-05	4.71E-05	2.14E-05	7.69E-05
21	1.81E-05	8.57E-06	8.40E-04	3.03E-03	1.22E-05	3.00E-05	9.19E-05	1.90E-04

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Table 2. *A CAS wavelets, Galerkin finite element method and the exact solution for* $\upsilon(\chi,\hat{t})$, $\sigma(\chi,\hat{t})$ *when* $\hat{t}$ *=9.5,* $\hat{t}$ *=13.5 and comparison between them* $\upsilon_0(\chi)=\sigma_0(\chi)=\sin^2\left(\frac{12}{10}\pi\chi\right)+\sin^2\left(\frac{-1}{10}\pi\chi\right)$

NO.	CAS Wavelets $\upsilon(\chi,\hat{t})$ $\hat{t}$=14.5	Galerkin Finite element method $\upsilon(\chi,\hat{t})$, $\hat{t}$=14.5	Exact Solution $\upsilon(\chi,\hat{t})$ $\hat{t}$=14.5	CAS wavelets $\sigma(\chi,\hat{t})$ $\hat{t}$=8	Galerkin Finite element method $\sigma(\chi,\hat{t})$, t=8	Exact Solution $\sigma(\chi,\hat{t})$ $\hat{t}$=8
1	0.01676753	0.01695454	0.01594873	0.72826192	0.77554676	0.76443563
2	0.01667693	0.01677521	0.01593418	0.79753859	0.81768448	0.80657393
3	0.01629418	0.01632946	0.01591805	0.97969854	0.91319185	0.98020868
4	0.01581375	0.01584698	0.01573587	1.00295773	1.00260765	1.00914925
5	0.01552882	0.01557386	0.01546253	1.04908083	1.04482057	1.05170986
6	0.01560161	0.01563745	0.01539631	1.02656122	1.02619394	1.02908211
7	0.01593843	0.01597543	0.01536432	0.95641845	0.95602083	0.95890937
8	0.01655065	0.01637400	0.01596299	0.86701762	0.86014518	0.89034389
9	0.01677542	0.01658927	0.01597819	0.80473367	0.80048434	0.80993732
10	0.01666415	0.01647741	0.01586636	0.81711765	0.81935633	0.81824762
11	0.01594643	0.01606683	0.01590502	0.90812098	0.90468775	0.90734775
12	0.01573507	0.01554364	0.01543203	1.00167385	1.00161710	1.00190506
13	0.01522607	0.01515988	0.01504874	1.06219537	1.06259648	1.06214853
14	0.01517993	0.01510713	0.01499600	1.06779761	1.06755464	1.06776443
15	0.01558367	0.01542225	0.01559109	1.01728809	1.01715799	1.01760468
16	0.01577813	0.01598064	0.01586953	0.93782376	0.93018756	0.95390674
17	0.01600137	0.01657747	0.01646632	0.85042574	0.84587901	0.85347679
18	0.01690879	0.01704017	0.01692906	0.80745692	0.80700317	0.80799209
19	0.01730477	0.01730286	0.01728175	0.82319381	0.82317497	0.82520638
20	0.01734002	0.01740520	0.01730599	0.88183075	0.88141190	0.88203098
21	0.01739987	0.01742652	0.01741795	0.87604015	0.87991302	0.87688019

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***Table 3**. The Absolute error for CAS wavelets, Galerkin finite element method with the exact solution for $\upsilon(\chi,\hat{t})$, $\sigma(\chi,\hat{t})$ when $\hat{t}$ =14.5, $\hat{t}$ =8., $\hat{t}$ =13.5, $\hat{t}$ =9.5*

NO.	CAS Wavelets $\upsilon(\chi,\hat{t})$ $\hat{t}$=9,5	Galerkin Finite element method $\upsilon(\chi,\hat{t})$,t=9,5	Exact Solution $\upsilon(\chi,\hat{t})$ $\hat{t}$=9,5	CAS Wavelets $\sigma(\chi,\hat{t})$ $\hat{t}$=13.5	Galerkin Finite element method $\sigma(\chi,\hat{t})$, $\hat{t}$=13,5	Exact Solution $\sigma(\chi,\hat{t})$ $\hat{t}$=13,5
1	0.99947887	0.99945457	0.99955488	0.04189611	0.04110720	0.04190700
2	0.99949230	0.99947123	0.99957116	0.04243655	0.04159035	0.04269024
3	0.99953581	0.99951240	0.99964123	0.04377317	0.04368605	0.04398658
4	0.99955963	0.99955657	0.99964564	0.04520569	0.04507065	0.04537045
5	0.99959187	0.99958132	0.99967812	0.04599862	0.04578861	0.04608850
6	0.99954155	0.99957533	0.99964752	0.04577914	0.04560786	0.04600775
7	0.99953912	0.99954407	0.99964439	0.04473945	0.04469978	0.04489967
8	0.99953614	0.99950661	0.99964065	0.04363260	0.04364617	0.04384606
9	0.99950048	0.99948520	0.99963851	0.04391755	0.04312346	0.04352335
10	0.99952576	0.99949328	0.99960931	0.04394595	0.04323447	0.04353436
11	0.99958324	0.99952845	0.99958845	0.04471332	0.04468572	0.04498561
12	0.99959233	0.99957422	0.99960741	0.04640536	0.04632790	0.04692779
13	0.99964551	0.99960918	0.99965090	0.04761359	0.04743364	0.04793353
14	0.99964657	0.99961824	0.99965181	0.04774001	0.04755889	0.04795878
15	0.99960512	0.99959954	0.99959994	0.04668060	0.04659355	0.04689344
16	0.99954245	0.99956406	0.99954639	0.04483311	0.04477505	0.04507494
17	0.99953946	0.99952862	0.99954285	0.04284865	0.04277671	0.04317660
18	0.99953736	0.99950634	0.99954062	0.04128423	0.04110823	0.04220812
19	0.99953650	0.99950015	0.99954000	0.04036414	0.04020614	0.04123103
20	0.99953644	0.99950323	0.99955032	0.03997833	0.03992289	0.03999976
21	0.99952353	0.99950576	0.99953576	0.03988845	0.03979043	0.03998032

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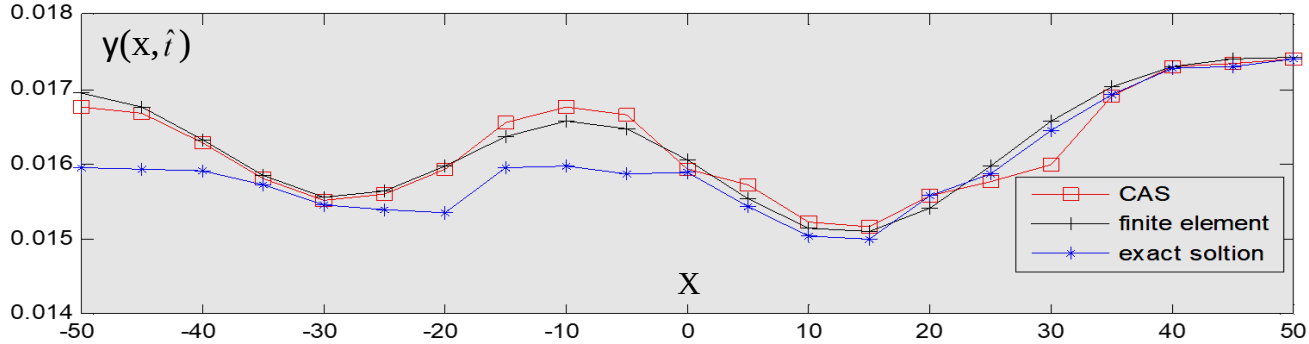

Figure 1. Comparison between CAS Galerkin finite elements (obtined from (Eq 11),(Eq 28) and Table 1) with exact solution where X={-50 ,-45 ,…., 45, 50}, Y=y(X, $\hat{t}$), L=0.5 $\Delta\hat{t} = 0.1, \varepsilon_1 = \varepsilon_2 = 0.001$, $\theta = 0.5, \alpha_1 = 1, \alpha_2 = 0.06,\ \beta_1 = \delta_1 = \delta_2 = 1$ and $\hat{t}$ =14.4

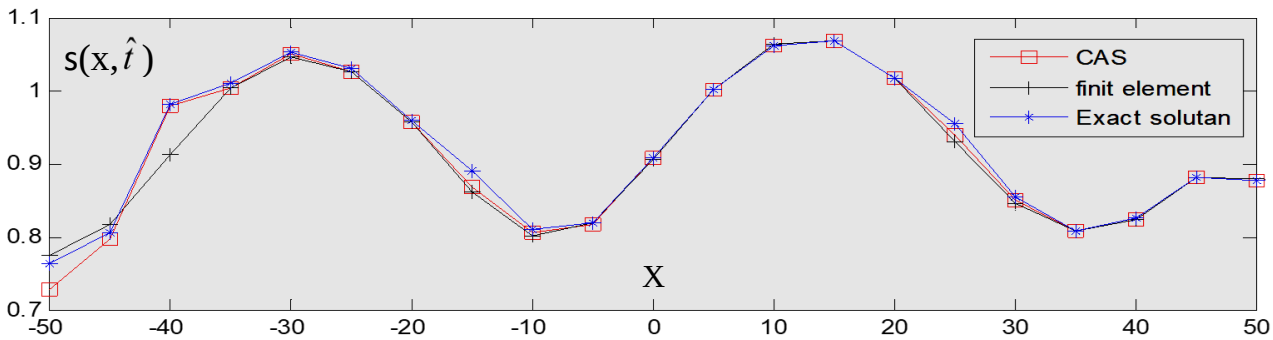

Figure 2. Comparison between CAS Galerkin finite elements (obtined from (Eq 11),(Eq 28)) and Table 1) with exact solution where X={-50 ,-45 ,…., 45, 50}, Y=s(X, $\hat{t}$), L=0.5 $\Delta\hat{t} = 0.1, \varepsilon_1 = \varepsilon_2 = 0.001$, $\theta = 0.5, \alpha_1 = 1, \alpha_2 = 0.06,\ \beta_1 = \delta_1 = \delta_2 = 1$ and $\hat{t}$ =8

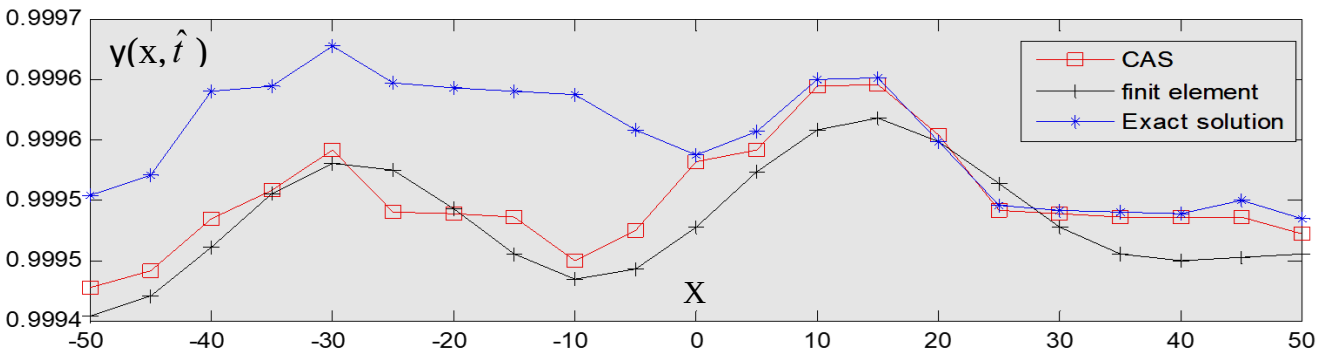

Figure 3. Comparison between CAS Galerkin finite elements (obtined from (Eq 11),(Eq 28)) and Table 1) with exact solution where X={-50 ,-45 ,…., 45, 50}, Y=y(X, $\hat{t}$), L=0.5, $\theta = 0.5$, $\alpha_1 = 1$, $\Delta\hat{t} = 0.1, \varepsilon_1 = \varepsilon_2 = 0.001$, , $\alpha_2 = 0.01$, $\beta_1 = \delta_1 = \delta_2 = 0.0001$, M=21 and $\hat{t}$ =9.5

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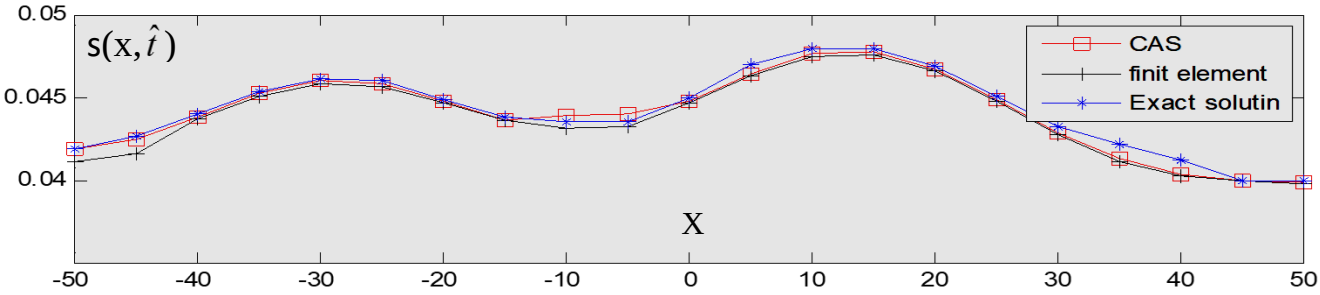

Figure 4. Comparison between CAS Galerkin finite elements (obtined from (Eq 11),(Eq 28)) and Table 1) with exact solution where X={-50 ,-45 ,…., 45, 50}, Y=S(X, $\hat{t}$), L=0.5, $\theta = 0.5$, $\alpha_1 = 1$, $\Delta\hat{t} = 0.1, \varepsilon_1 = \varepsilon_2 = 0.001$, , $\alpha_2 = 0.01$, $\beta_1 = \delta_1 = \delta_2 = 0.0001$, M=21 and $\hat{t}$ =13.5

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