PERIÓDICO TCHÊ QUÍMICA
ARTIGO ORIGINAL

ESTIMATIVA DE ERRO DO ALGORITMO PARA A DEFINIÇÃO DO DESLOCAMENTO DE FASE DOS SINAIS HARMÔNICOS NO TEMPO MENOR QUE O PERÍODO DO SINAL UTILIZANDO AMOSTRAGEM ESTOCÁSTICA

ERROR ESTIMATION OF THE ALGORITHM FOR THE PHASE SHIFT DEFINITION OF HARMONIC SIGNALS IN THE TIMELESS THAN THE SIGNAL PERIOD USING STOCHASTIC SAMPLING

ОЦЕНКА ПОГРЕШНОСТЕЙ АЛГОРИТМА ОПРЕДЕЛЕНИЯ СДВИГА ФАЗ ГАРМОНИЧЕСКИХ СИГНАЛОВ ЗА ВРЕМЯ, МЕНЬШЕЕ ПЕРИОДА, С ИСПОЛЬЗОВАНИЕМ СТОХАСТИЧЕСКОЙ ДИСКРЕТИЗАЦИИ

ZAITSEVA, Irina N.[1*]

[1] Bunin Yelets State University, Department of Physics, Radio Engineering and Electronics, Russia.

** Corresponding author*
e-mail: irina.n.zaitseva@yandex.ru

Received 23 June 2020; received in revised form 27 July 2020; accepted 07 August 2020

RESUMO

A determinação dos parâmetros de um sinal harmônico é um dos tipos mais comuns de medições em engenharia de rádio, engenharia de comunicação, eletrônica e sistemas de automação. A pesquisa e o desenvolvimento de novos métodos para medir os parâmetros dos sinais harmônicos são relevantes. Este trabalho estudou os erros de algoritmo para determinar a mudança de fase dos sinais harmônicos utilizando amostragem estocástica. A relevância deste estudo é ditada pelo aumento dos requisitos de precisão e velocidade dos equipamentos de medição, a redução do tempo necessário para decidir sobre a presença de um sinal enquanto se procura por ele e o que torna necessário utilizar métodos estatisticamente ótimos para medir os parâmetros de sinais. O objetivo do trabalho foi desenvolver um algoritmo e estimar seus erros para a possibilidade de implementação prática do algoritmo para o processamento de sinais de rádio de freqüência infra baixa durante a amostragem estocástica. Os valores instantâneos em cada amostra dos sinais sob investigação são baseados na amostragem estocástica no tempo, de acordo com a lei de distribuição uniforme. A modelagem matemática dos erros do algoritmo para determinar o deslocamento de fase dos sinais com harmônicas e, dependendo das harmônicas em comparação com a primeira (principal) harmônica do sinal sob investigação durante a amostragem por conversores analógico-digitais reais, foram realizados. Os valores obtidos dos erros do algoritmo para determinar o deslocamento de fase do harmônico principal estão dentro de uma faixa aceitável (<30%); em amplitudes harmônicas (até o 3º harmônico), dentro de 20%. Os resultados do experimento de computação para estimar os erros do algoritmo confirmam a possibilidade de obter alta precisão na determinação do deslocamento de fase dos sinais harmônicos. Este algoritmo pode ser usado para processar sinais de rádio de freqüência infrabaixa com precisão suficiente em acústica, hidroacústica, acústica sísmica, comunicação subaquática e subterrânea.

Palavras-chave: *tempo de acesso, modelagem computadorizada, sinal de rádio de freqüência infrabaixa, conversor analógico-digital.*

ABSTRACT

Determining the parameters of a harmonic signal is one of the most common types of measurements in radio engineering, communication engineering, electronics and automation systems. The research and development of new methods for measuring the harmonic signal parameters are relevant. This work studied algorithm errors for determining the phase shift of harmonic signals using stochastic sampling. The relevance of this study is dictated by increasing requirements for the accuracy and speed of measuring equipment, the reduction of time it takes to decide on the presence of a signal while searching for it, that make it necessary to use statistically optimal methods for measuring signal parameters. The work aimed to develop an algorithm and estimate its errors for the possibility of practical implementation of the algorithm for processing infra-low-frequency radio signals during stochastic sampling. According to the uniform distribution law, the instantaneous values in each sample of the signals under investigation are based on stochastic sampling in time.

DOI: 10.52571/PTQ.v17.n36.2020.229_Periodico36_pgs_213_222.pdf

Mathematical modeling of algorithm errors for determining the phase shift of signals with harmonics, and depending on harmonics compared to the first (main) harmonic of the signal under investigation during the sampling by real analog-to-digital converters have been carried out. The obtained values of the algorithm errors for determining the phase shift of the main harmonic are within an acceptable range (<30%); at harmonics amplitudes (up to the 3rd harmonic) within 20%. The computing experiment results for estimating the algorithm errors confirm the possibility of obtaining high accuracy in determining the phase shift of harmonic signals. This algorithm can be used for processing infra-low-frequency radio signals with sufficient accuracy in acoustics, hydroacoustics, seismic acoustics, underwater, and underground communication.

Keywords: *access time, computer modeling, infra-low frequency radio signal, analog-to-digital converter.*

АННОТАЦИЯ

Определение параметров гармонического сигнала является одним из самых распространенных видов измерений в радиотехнике, технике связи, электронике и системах автоматики. Исследование и разработка новых методов измерения параметров гармонического сигнала являются актуальными. Настоящая работа посвящена исследованию погрешностей алгоритма определения сдвига фаз гармонических сигналов, с использованием стохастической дискретизации. Актуальность данного исследования продиктована возрастающими требованиями к точности и быстродействию измерительной аппаратуры, сокращению времени принятия решения о наличии сигнала при его поиске, делают необходимым использование статистически оптимальных методов измерения параметров сигнала. Цель работы заключается в разработке алгоритма и оценке его погрешностей для возможности практической реализации алгоритма для обработки инфранизкочастотных радиосигналов при стохастической дискретизации. Мгновенные значения в каждой выборке из исследуемых сигналов основываются на стохастической дискретизации во времени по равномерному закону распределения. Проведено метаматематическое моделирование погрешностей алгоритма определения сдвига фаз сигналов с гармониками и в зависимости от гармоник относительно первой (основной) гармоники исследуемого сигнала при дискретизации реальными аналого-цифровыми преобразователями. Полученные значения погрешностей алгоритма определения сдвига фаз основной гармоники находятся в приемлемом диапазоне (<30%); при амплитудах гармоник (не далее 3-й гармоники) – до 20%. Результаты вычислительного эксперимента оценки погрешностей алгоритма подтверждают возможность получения высокой точности определения сдвига фаз гармонических сигналов. Настоящий алгоритм может найти применение при обработке инфранизкочастотных радиосигналов с достаточной точностью в акустике, гидроакустике, сейсмоакустике, подводной и подземной связи.

Ключевые слова: *время обращения, математическое моделирование, инфранизкочастотный радиосигнал, аналого-цифровой преобразователь.*

1. INTRODUCTION:

The radio signals currently used in radio engineering systems have a wide variety of shapes and types. For example, signals with complex modulation are used in satellite and ground-based navigation and communications systems. More straightforward signals are used, for instance, in radiolocation, acoustics, and medicine. However, a harmonic signal is most often the basis of their carrier signal. The presence of the harmonic signal makes it possible to build high-precision measuring systems using one of the parameters of this signal – its phase (Shakhov and Ugol'kov, 1986; Ugol’kov, 2003, 2004; Zaitseva, 2019).

Therefore the research and development of new methods for measuring harmonic signal parameters are relevant. It is also appropriate for developing high-precision radio engineering systems, measuring equipment, and the simpler systems that use harmonic signal parameters as information ones (geodesy, acoustics, and medicine) Marmarelis and Marmarelis, 1978; Levin, 1989; Bilinsky and Mikelson, 1983).

One of the main criteria for the quality of the radio system is the time of the measurement result issue from the moment when the signal is received at its input. This time depends, among other factors, also on the measurement (estimation) time of the parameters of the carrier harmonic signal. This is especially important for systems and devices that use sufficiently low-frequency radio signals, pulsed signals with harmonic filling. Also, for devices that output low-frequency signals during the processing of high-frequency signals (for example, signals containing Doppler frequency shift, in radiolocation, and the receiving and measuring equipment of satellite navigation systems). Reduction of the time for the definition of the main parameters of harmonic signals such as amplitude, frequency, phase (phase shift) and spectrum based on various basic functions to the minimum values of access time to physical signals (at least much shorter than their period of existence by dozens of times and more), as well as the development of fundamental analog-digital algorithms corresponding to this goal, is a very relevant problem, according to paper (Zaitseva, 2019; Meshkov and Ugol'kov, 1984; Shakhov and Ugol'kov 1986).

It is necessary to reduce the access time to the signals understudy to less than one signal period to measure the parameters of signals in acoustics, hydroacoustics, and seismic acoustics. This is because the periods of infra-low-frequency signals can be equal to days, months, years or more. This raises the question of determining the minimum access time to the signal (minimum instantaneous counts) and determining the parameters of harmonic signals for a time significantly less than their period. In scientific works (Shakhov and Ugol'kov, 1986; Ugol’kov, 2003, 2004; Meshkov and Ugol'kov, 1984; Zaitseva, 2019), the issues of determining the main parameters of harmonic signals by the minimum of instantaneous readings for a time less than their period are considered.

Algorithms for defining the parameters of harmonic signals for the timeless than their period, using "classical" methods of uniform time sampling of these signals are considered in papers (Shakhov and Ugol'kov, 1986; Ugol’kov, 2003, 2004). Of particular interest are the studies (Zaitseva, 2019; Porschnev and Kussaikin, 2016) of the application in such algorithms of stochastic sampling for optimizing the algorithms under conditions of the definition of the real signals parameters with distortions, external noise, and self-noise.

Fundamental interest in such signal processing appears in the analysis of transmission lines, in which signals modulated by phase, frequency, or amplitude are encoded by conversion to "solitons." A soliton is a structurally stable solitary wave propagating in the nonlinear medium. Solitons behave like particles (wave-particle duality), and when interacting with each other or with some other disturbances, they are not destroyed but continue to move, keeping their structure. This transmission line usually consists of a large number of nonlinear chains. It can be described by a partial differential system that is transformed into the Korteweg-de Vries equation.

Therefore, the development and analysis of analog-to-digital algorithms for the definition of the parameters of harmonic low-frequency signals in the time much less of their period to both reduce the time for detecting processes and objects, and to predict critical phenomena in hydroacoustics, acoustics and thermal (wave) processes is a very relevant problem (Gorbunov, 2014; Amelin and Granichin, 2011; Marmarelis and Marmarelis, 1978; Porschnev and Kussaikin, 2016; Boashash, 2015; Rajan, 2017; Taylor, 2017; Ermolaeva, Goncharenko, and Gordienko, 2011; Chaparro, 2015; Chapman, 2017).

Thus, the study aimed to estimate the algorithm errors for determining the phase shift of harmonic signals in the time shorter than the period for a possible practical implementation of the algorithm in real conditions at underwater objects.

2. MATERIALS AND METHODS:

The problem of reducing the access time to signals to determine their parameters has always been considered in the form of a statement since the emergence of the theory of signal processing in electrical engineering, measurement technology, radio engineering, and numerous practical applications of this branch of science and technology. Since the early nineties, these studies have been further stimulated and developed with the advent of high-speed and small-size computing technology, new software, and the possibility of accessing it by a wide range of scientists and engineers.

Studies of digital methods for determining the parameters of harmonic signals by measuring their instantaneous values in a timeless than a period using analog-to-digital processing and high-speed ADCs are of great interest. Currently, various methods of digital processing of radio signals are used. Some examples are the orthogonal transformations, modeling, and digital filtering methods, an analytical apparatus for describing signals and noise, numerical methods of solving problems and systems of equations. (Bilinsky and Mikelson, 1983; Wentzel, 1969; Meshkov and Ugol'kov, 1984; Gorbunov, Kulikov, and Shpak, 2016).

In (Zaitseva, 2019), based on solving the system of equations, an algorithm for determining the phase shift of harmonic signals was developed. The error estimation of the developed phase-shift algorithm for harmonic signals was studied with the probabilistic and statistical method using stochastic sampling in this work.

The basic relations will be presented for the algorithm for the phase shift definition of harmonic signals in access time less than one signal period.

It will be assumed that the reference *x(t)* and the investigated *y(t)* signals, along with the harmonic component, have constant components C_{0x} and C_{0y}, respectively. Equations 1-6 show their instantaneous sample values and finite differences:

$$\begin{cases} x_0 = A_{mx} \cdot \sin(\omega \cdot t) + C_{0x}; \\ x_1 = A_{mx} \cdot \sin(\omega \cdot t + h) + C_{0x}; \\ x_2 = A_{mx} \cdot \sin(\omega \cdot t + 2 \cdot h) + C_{0x}; \end{cases} \quad \text{(Eq. 1)}$$

$$\begin{cases} \Delta_{1x} = x_1 - x_0 = 2 \cdot \sin(h/2) \cdot \cos(\omega \cdot t + h/2); \\ \Delta_{2x} = x_2 - x_1 = 2 \cdot \sin(h/2) \cdot \cos(\omega \cdot t + h \cdot 3/2); \end{cases} \quad \text{(Eq. 2)}$$

$$\Delta_{ix} = x_{i+1} - x_i = 2 \cdot \sin(h/2) \cdot \cos(\omega \cdot t + i \cdot h/2); \quad \text{(Eq. 3)}$$

$$\begin{cases} y_0 = A_{my} \cdot \sin(\omega \cdot t + \varphi) + C_{0y}; \\ y_1 = A_{my} \cdot \sin(\omega \cdot t + \varphi + h) + C_{0y}; \\ y_2 = A_{my} \cdot \sin(\omega \cdot t + \varphi + 2 \cdot h) + C_{0y}; \end{cases} \quad \text{(Eq. 4)}$$

$$\begin{cases} \Delta_{1y} = y_1 - y_0 = 2 \cdot \sin(h/2) \cdot \cos(\omega \cdot t + \varphi + h/2); \\ \Delta_{2y} = y_2 - y_1 = 2 \cdot \sin(h/2) \cdot \cos(\omega \cdot t + \varphi + h \cdot 3/2); \end{cases} \quad \text{(Eq. 5)}$$

$$\Delta_{iy} = y_{i+1} - y_i = 2 \cdot \sin(h/2) \cdot \cos(\omega \cdot t + \varphi + i \cdot h/2), \quad \text{(Eq. 6)}$$

where: ω is the frequency of the harmonic components of signals *x(t)* and *y(t)*; φ is the phase shift between the harmonic components of signals *x(t)* and *y(t)*; $h = \omega{\cdot}\Delta t$ is the sampling step in angular measurement; Δt is the sampling interval of signals *x(t)* and *y(t)* over time; $i = 0, 1, 2, \ldots, n$.

Having found the relations

$$J_1 = \frac{\Delta_{1x}}{\Delta_{2x}} = \frac{\cos(h/2) - \mathrm{tg}(\alpha_0) \cdot \sin(h/2)}{\cos(h \cdot 3/2) - \mathrm{tg}(\alpha_0) \cdot \sin(h \cdot 3/2)}; \quad \text{(Eq. 7)}$$

$$J_2 = \frac{\Delta_{1y}}{\Delta_{2x}} = \frac{\cos(h/2) - \mathrm{tg}(\varphi) \cdot \sin(\alpha_0 + h/2)}{\cos(\alpha_0 + h \cdot 3/2) - \mathrm{tg}(\varphi) \cdot \sin(\alpha_0 + h \cdot 3/2)}, \quad \text{(Eq. 8)}$$

and having solved Equations 7 and 8 for φ and α_0, it is obtained the expression for the definition of the phase shift φ along the curve of instantaneous values of the first order finite differences Δ_{ix}и Δ_{iy}:

$$\varphi = \mathrm{arctg}\frac{J_2 \cos(\alpha_0 + h \cdot 3/2) - \cos(\alpha_0 + h/2)}{J_2 \sin(\alpha_0 + h \cdot 3/2) - \sin(\alpha_0 + h/2)}, \quad \text{(Eq. 9)}$$

where

$$\alpha_0 = \mathrm{arctg}\frac{J_1 \cos(h \cdot 3/2) - \cos(h/2)}{J_1 \sin(h \cdot 3/2) - \sin(h/2)}.$$

The phase shift value φ obtained by the algorithm (9) does not depend on the constant components C_{0x} and C_{0y} and the amplitudes A_{m1} and A_{m2} of *x(t)* and *y(t)* signals.

The algorithm (9) allows for the definition of the phase shift with minimum access time to signals τ equal to $\tau = 2{\cdot}\Delta t$ in the angular measure 2*h*.

Stochastic processing of harmonic signals (1) and (4) to define the phase shift is that the random law carries out the sampling of instantaneous values from the signal. The sampling interval of signals (1) and (4) overtime is selected from uniformly distributed linearly transformed random values, arranged as an ordered series, according to papers (Boashash, 2015; Rajan, 2017; Taylor, 2017).

It is assumed that the uniform law distributes samples from signals (1) and (4). The expression describes the density of time intervals distribution for discrete samples:

$p(t_i) = 1/T_m$ at $t_1 \leq t_i \leq t_n$,

where $T_m = t_n - t_1$ is the measurement time or access time to the signal, and

$p(t_i) = 0$ in case of other t_i. (Eq. 10)

It is determined the distribution density for $x(t_i)$ at t_i uniformly distributed in the range from t_1 до t_n, as a function of *x* according to papers (Wentzel, 1969; Levin, 1989):

$$p(x,\ t_i) = \frac{1}{\omega \cdot t_m}.$$

To define the phase shift of harmonic signals according to the algorithm (9), it was used the formula for the statistical expectation of values of the signals under consideration with stochastic sampling over time as a stochastic process. The constant components of the signals may not be taken into account, because they are reduced when finite differences of the first order are found (1)-(8). Taking into account Equations 10 and 11, the statistical expectation for the signal (1) is expressed by the formula:

$$m_{1x}(x,\ t_i) = \int_{x_1}^{x_n} x \cdot p(x,\ t_i) \cdot dx = \frac{2 \cdot A \cdot \sin(\omega \cdot T_m / 2)}{\omega^2 \cdot T_m} \cdot \sin(2 \cdot \alpha_0 + \omega \cdot T_m / 2),$$

where

$$x_1 = A \cdot \sin(\alpha_0);$$

$$x_n = A \cdot \sin(\alpha_0 + \omega \cdot T_m).$$

For the second sample from the signal (1), the statistical expectation, according to paper (Wentzel, 1969) is:

$$m_{2x}(x,\ t_i) = \int_{x_n}^{x_{n+1}} x \cdot p(x,\ t_i) \cdot dx = \frac{2 \cdot A \cdot \sin(\omega \cdot T_m / 2)}{\omega^2 \cdot T_m} \cdot \sin(2 \cdot \alpha_0 + 3 \cdot \omega \cdot T_m / 2),$$

where

$$x_{n+1} = A \cdot \sin(\alpha_0 + 2 \cdot \omega \cdot T_m).$$

For the third sample from the signal (1), the statistical expectation is:

$$m_{3x}(x,\ t_i) = \int_{x_{n+1}}^{x_{n+2}} x \cdot p(x,\ t_i) \cdot dx = \frac{2 \cdot A \cdot \sin(\omega \cdot T_m / 2)}{\omega^2 \cdot T_m} \cdot \sin(2 \cdot \alpha_0 + 5 \cdot \omega \cdot T_m / 2),$$

where

$$x_{n+1} = A \cdot \sin(\alpha_0 + 3 \cdot \omega \cdot T_m).$$

Using similar Equations 10 and 11, the expression for the signal (4) is:

$$m_{1y}(y,\ t_i) = \int_{y_1}^{y_n} y \cdot p(y,\ t_i) \cdot dy = \frac{2 \cdot A \cdot \sin(\omega \cdot T_m / 2)}{\omega^2 \cdot T_m} \cdot \sin(\varphi + 2 \cdot \alpha_0 + \omega \cdot T_m / 2)$$

where

$$y_1 = A \cdot \sin(\varphi + \alpha_0);$$

$$y_n = A \cdot \sin(\varphi + \alpha_0 + \omega \cdot T_m).$$

Calculating the statistical expectation for the second sample from the signal (4), according to paper (Wentzel, 1969):

$$m_{2y}(y,\ t_i) = \int_{y_n}^{y_{n+1}} y \cdot p(y,\ t_i) \cdot dy = \frac{2 \cdot A \cdot \sin(\omega \cdot T_m / 2)}{\omega^2 \cdot T_m} \cdot \sin(\varphi + 2 \cdot \alpha_0 + 3 \cdot \omega \cdot T_m / 2),$$

where

$$y_{n+1} = A \cdot \sin(\varphi + \alpha_0 + 2 \cdot \omega \cdot T_m).$$

The expression describes the statistical expectation for the second sample from the signal (4):

$$m_{3y}(y,\ t_i) = \int_{y_{n+1}}^{y_{n+2}} y \cdot p(y,\ t_i) \cdot dy = \frac{2 \cdot A \cdot \sin(\omega \cdot T_m / 2)}{\omega^2 \cdot T_m} \cdot \sin(\varphi + 2 \cdot \alpha_0 + 5 \cdot \omega \cdot T_m / 2),$$

where

$$y_{n+1} = A \cdot \sin(\varphi + \alpha_0 + 3 \cdot \omega \cdot T_m).$$

To find the finite differences of the first order from these samples:

$$\Delta_{1x} = m_{2x} - m_{1x} = k \cdot 2 \cdot \sin(\omega \cdot T_m / 2) \cdot \cos(\alpha_0 + \omega \cdot T_m);$$

$$\Delta_{2x} = m_{3x} - m_{2x} = k \cdot 2 \cdot \sin(\omega \cdot T_m / 2) \cdot \cos(\alpha_0 + 2 \cdot \omega \cdot T_m);$$

$$\Delta_{1y} = m_{2y} - m_{1y} = k \cdot 2 \cdot \sin(\omega \cdot T_m / 2) \cdot \cos(\varphi + 2 \cdot \alpha_0 + \omega \cdot T_m);$$

$$\Delta_{2y} = m_{3y} - m_{2y} = k \cdot 2 \cdot \sin(\omega \cdot T_m / 2) \cdot \cos(\varphi + 2 \cdot \alpha_0 + 2 \cdot \omega \cdot T_m),$$

where

$$k = \frac{2 \cdot A \cdot \sin(\omega \cdot T_m / 2)}{\omega^2 \cdot T_m}.$$

Having found the relations

$$J_1 = \frac{\Delta_{1x}}{\Delta_{2x}} = \frac{\cos(2 \cdot \omega \cdot T_m) - \mathrm{tg}(\alpha_0) \cdot \sin(\omega \cdot T_m)}{\cos(2 \cdot \omega \cdot T_m) - \mathrm{tg}(\alpha_0) \cdot \sin(2 \cdot \omega \cdot T_m)}; \text{ (Eq. 12)}$$

$$J_2 = \frac{\Delta_{1y}}{\Delta_{2y}} = \frac{\cos(2\cdot\alpha_0 + \omega\cdot T_m) - \mathrm{tg}(\varphi)\cdot\sin(2\cdot\alpha_0 + \omega\cdot T_m)}{\cos(2\cdot\alpha_0 + 2\cdot\omega\cdot T_m) - \mathrm{tg}(\varphi)\cdot\sin(2\cdot\alpha_0 + 2\cdot\omega\cdot T_m)}, \quad \text{(Eq. 13)}$$

and having solved Equations 12 and 13 for φ and α_0, it was obtained the expression for the definition of the phase shift φ along the curve of instantaneous values of the first order finite differences Δ_{ix} and Δ_{iy}:

$$\varphi = \mathrm{arctg}\frac{J_2\cos(2\cdot\alpha_0 + 2\cdot\omega\cdot T_m) - \cos(2\cdot\alpha_0 + \omega\cdot T_m)}{J_2\sin(2\cdot\alpha_0 + 2\cdot\omega\cdot T_m) - \sin(2\cdot\alpha_0 + \omega\cdot T_m)}, \quad \text{(Eq. 14)}$$

where

$$\alpha_0 = (\mathrm{arctg}\frac{J_1\cos(2\cdot\omega\cdot T_m) - \cos(\omega\cdot T_m)}{J_1\sin(2\cdot\omega\cdot T_m) - \sin(\omega\cdot T_m)})/2.$$

The phase shift value φ obtained by the algorithm (14) does not depend on the constant components C_{0x} and C_{0y} and the amplitudes A_{m1} and A_{m2} of *x(t)* and *y(t)* signals.

The relative error of the phase shift definition φ in this case is equal to:

$$\gamma_\varphi = \frac{\varphi' - \varphi}{\varphi} \cdot 100\% \quad \text{(Eq. 15)}$$

This algorithm can also be used for analyzing the effects of interference, even not a multiple of the frequency ω, on the error of the phase shift definition. For this purpose, it is necessary to select the corresponding k by limiting, for example, the signal (4) to the sum of any number of harmonic aliquant and multiple frequencies.

3. RESULTS AND DISCUSSION:

To estimate the errors of the presented algorithm for the phase shift definition of harmonic signals, harmonic signals in the form of an infra-low frequency sinusoidal wave with the frequency of the first harmonic *f* = 20 Hz and amplitude *A* = 10 *B* were studied, presented by the function:

$$x(t_i) = 10\cdot\sin(\omega\cdot t_i); \quad \text{(Eq. 16)}$$

$$y(t_i) = 10\cdot sin(\omega\cdot t_i + \varphi) + \frac{10}{g}\cdot sin(2\cdot\omega\cdot t_i + \varphi) + \frac{10}{g}\cdot sin(3\cdot\omega\cdot t_i + \varphi), \quad \text{(Eq. 17)}$$

where: $^{\wedge}t_i$ are the time intervals of the discrete sample and signal, distributed according to the uniform law;

ω is the circular frequency of the signal, rad/s;

φ = 0.5 rad is the phase shift;

$g = 10 ... 500$ is the coefficient.

The total number of discrete samples with uniform distribution is equal to 2000 for the signal period. The number of three discrete samples during the access time to the signal with access timeless than its period arbitrarily is equal to 300. Then, the access time to the period of signals under investigation will be less than the first harmonic period.

The numerical modeling was performed in *MathCAD* software package. Using the built-in statistical function *rnd*, it was generated 2000 numbers with a uniform distribution law. Further, it was arrange the obtained values in the form of an ordered series in ascending order and calculate the values of discrete samples from sinusoidal signals without harmonics and with two harmonics (even and odd). The uniformly distributed harmonic amplitude values from 1% to 20% of the amplitude of the main (first) harmonic are of interest. The modeling results are illustrated in Figures 1 and 2.

Figure 1 represents the reconstructed period of the signals under investigation in the form of sinusoid *x(t)* without harmonics and *y(t)* with two harmonics with stochastic sampling over time and with the uniform distribution law. The harmonics are distributed with the uniform amplitude equal to 20% of the amplitude of the main (first) harmonic. It can be observed that the signal *y(t)* is significantly distorted.

Figure 2 represents a diagram calculated compared to the phase shift error of harmonic signals where the signal *y(t)* has harmonics with an amplitude between 1% and 20% of the amplitude of the main (first) harmonic. The errors have been calculated for three samples in the form of statistical expectation of three samples with 300 samples each, synchronously from both signals with time sampling in the form of samples with uniform distribution law.

Table 1 represents the computational results of the phase shift errors of harmonic signals with harmonics in time less than one signal period with stochastic sampling over time.

At the moment, there are some works focused on the development and research of analog-digital algorithms for determining the parameters of infra-low-frequency radio signals for a shorter period both for special cases and for individual signal parameters. However, such developments are important to reduce the time of identification and detection of processes and objects and study high-speed objects and forecast critical phenomena in hydroacoustics, acoustics, and thermal (wave) processes.

Of particular interest are studies on applying stochastic sampling in such algorithms for the optimization of algorithms' operation under conditions of organized interference and noise. The obtained results can be used in solving the following problems of science and exclusive equipment:

– In experimental investigations of the infra-low frequency hydroacoustic and meteorological properties of the processes taking place in the seas and oceans (especially in the coastal areas), using installed underwater hydroacoustic antennas and presence of fields of thermal sensors;

– In the development of devices with accelerated hydroacoustic and thermal detection and identification of high-speed (slow-moving or motionless) and low-noise underwater objects;

– In the development of methods and devices for short-term and long-term forecasting of catastrophic wave events such as seismic sea waves, earthquakes and atmospheric phenomena (hurricanes and global wind vortices) since their emergence;

– For the metrological certification of hydroacoustic antennas and fields of thermal sensors;

– For the development of methods and devices for non-destructive control of the state of large facilities and structures such as bridges, large sports facilities, tunnels, pipelines to accelerate the critical (short-term), and long-range forecasting of their standard operating condition.

4. CONCLUSIONS:

The developed algorithm for determining the phase shift of harmonic signals uses the minimum number of instantaneous readings of the measured signals (three). It can be used when measuring infra-low-frequency radio signals. Instantaneous values in each sample of the signals under study are based on stochastic sampling in time according to a uniform distribution law with a signal access time of a shorter period.

The phase shift definition errors by the developed algorithm are fractions of a percent in the signal y(t) when the harmonic content is less than 1%. Further, the phase shift errors (accuracy) increase almost linearly from -2% to -27% when the amplitude (total) of the harmonics increases from 2% to 20% of the amplitude of the first harmonic. The phase shift errors of the algorithm under investigation with stochastic sampling are in an acceptable range of <30 % at the amplitude of harmonics (up to the third harmonic) within 20%.

The error in determining the phase shift of the investigated harmonic signals by the developed algorithm with stochastic sampling is hundredths of a percent and depends little on the signal sampling accuracy by level. The obtained error values correspond to the sampling accuracy when converting the analog-to-digital converters (ADCs) of 12-bit ADCs into the accepted values.

Thus, in the broad sense, the proposed research is one of the parties to the further development of this and other algorithms of similar signals and analysis of their errors.

5. REFERENCES:

1. Amelin, K.S. and Granichin, O.N. (2011). Randomization possibilities in prediction algorithms of the Kalman type for arbitrary external interferences in tracking. *Gyroscopy and Navigation, 2*(73), 38-50.

2. Bilinsky, I.Y. and Mikelson, A.K. (1983). *Stochastic digital processing of continuous signals*. Riga, Latvia: Zinatne.

3. Boashash, B. (2015). *Time-frequency signal analysis and processing: A comprehensive reference.* Elsevier.

4. Chaparro, L. (2015). Fourier analysis of discrete-time signals and systems. In *Signals and systems using MATLAB* (pp. 683-768). Elsevier.

5. Chapman, N.R. (2017). Inverse methods in underwater acoustics. In T.H. Neighbors, III and D. Bradley (Eds.), *Applied*

underwater acoustics (pp. 553-585). Elsevier.

6. Ermolaeva, E.O., Goncharenko, B.I. and Gordienko, V.A. (2011). Vector-phase methods as the basis for creation of new generation of measuring hydroacoustic systems. *Proceedings of Forum Acusticus,* 2819-2824.

7. Gorbunov, Yu.N. (2014). Stochastic radiolocation: conditions for the problem solving of detection, estimation and filtration. *Journal of Radio Electronics, 11*, 1-23.

8. Gorbunov, Yu.N., Kulikov, G.V. and Shpak, A.V. (2016). *Radiolocation: stochastic approach.* Moscow, Russia: Scientific and Technical Publishing House "Hot Line Telecom".

9. Levin, B.R. (1989). *Theoretical foundations of statistical radio engineering.* Moscow, Russia: Radio and Communication.

10. Marmarelis, P.Z. and Marmarelis, V.Z. (1978). *Analysis of physiological systems: The white-noise approach.* Boston, MA: Springer.

11. Meshkov, V.P. and Ugol'kov, V.N. (1984). *Definition of harmonic signal parameters with a minimum of instantaneous samples.* Krasnoyarsk, Russia: Publishing House of the L.V. Kirensky Institute of Physics of the Academy of Sciences of the USSR.

12. Porschnev, S.V. and Kussaikin, D.V. (2016). About the accuracy of extraction of the periodic discrete signal of the finit-duration by means of the Kotel'nikov series. *T-Comm: Telecommunications and Transportation, 10*(11), 4-8.

13. Rajan, D. (2017). Probability, random variables, and stochastic processes. In E. Serpedin, T. Chen and D. Rajan (Eds.), *Mathematical foundations for signal processing, communications, and networking* (pp. 205-244). Boca Raton, FL: CRC Press.

14. Shakhov, E.K. and Ugol'kov, V.N. (1986). *The question of determining the phase shift of harmonic signals in the time less than the signal period in the presence of the constant component.* Krasnoyarsk, Russia: Publishing House of the L.V. Kirensky Institute of Physics of the Academy of Sciences of the USSR.

15. Taylor, F.J. (2017). Digital signal processing. In V.G. Oklobdzija (Ed.), *Digital systems and applications*. Boca Raton, FL: CRC Press.

16. Ugol’kov, V.N. (2003). Methods of measuring the phase shift and amplitude of harmonic signals using integral samples. *Measurement Techniques, 46*, 495-501.

17. Ugol’kov, V.N. (2004). Some problems of the digital analysis of signal spectra. *Measurement Techniques, 47*(6), 601-606.

18. Wentzel, E.S. (1969). *Probability theory.* Moscow, Russia: Science.

19. Zaitseva, I.N. (2019, 1-3 July). *Some problems of discretization and determination of basic parameters of harmonic signals.* Paper presented at the Conference “Systems of Signal Synchronization, Generating and Processing in Telecommunications (SYNCHROINFO)”. doi: 10.1109/SYNCHROINFO.2019.8814150

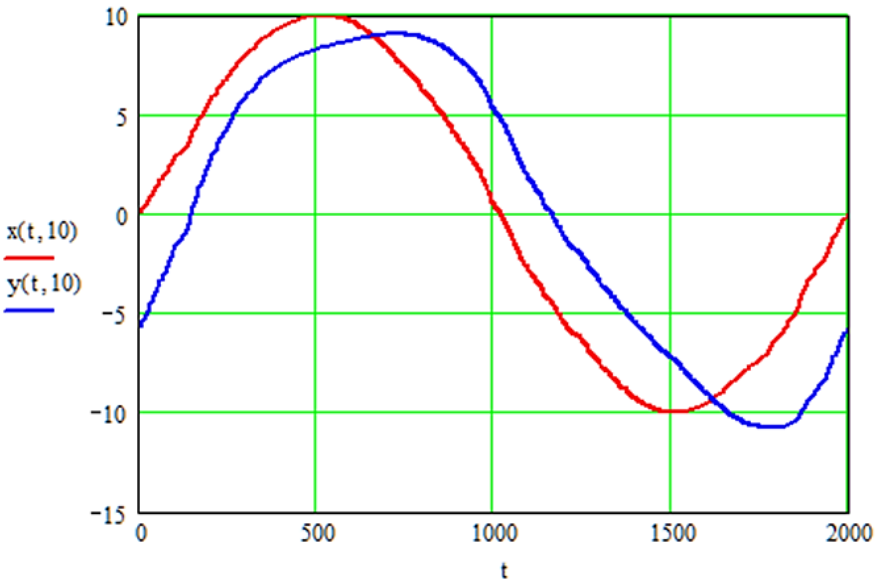

Figure 1. *The reconstructed period of the signals under investigation $x\ (t)$ and $y(t)$ with the harmonics' amplitude equal to 20%. Source: the author*

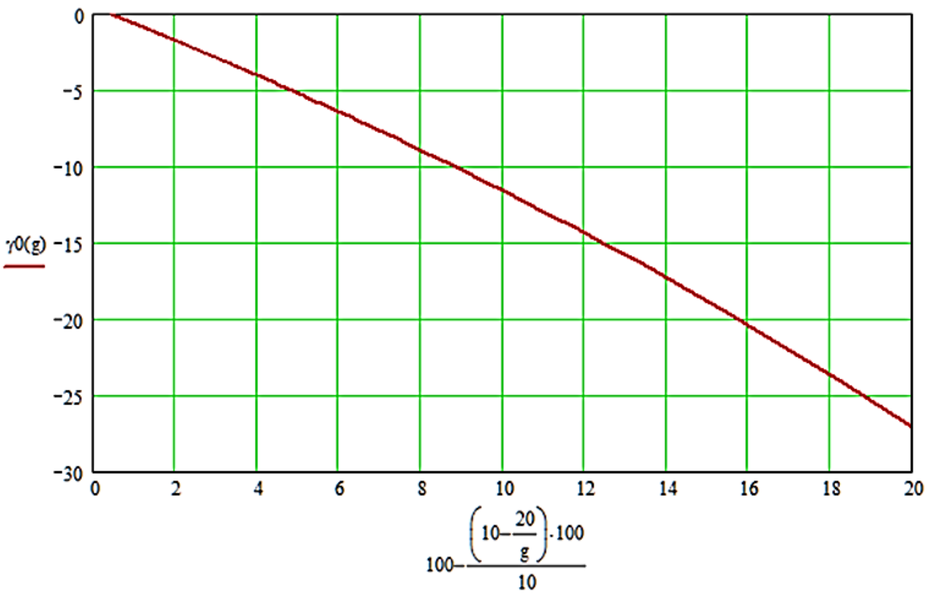

Figure 2. *Errors of defining the phase shift of the signals under investigation (X-axis, %) compared to the harmonics in the signal y(t) (Y-axis, %). Source: the author*

Table 1. _Computational results of the phase shift errors_

Harmonic percentage (two harmonics), %	**The relative error of phase shift detection using stochastic sampling on instantaneous samples with uniform distribution, %**
1	0
2	-2
5	-5
10	-12
14	-17
18	-24
20	-27